

thm_2Epath_2Efinite__path__end__cases
(TMGxyZzHWYUF-
sEUTw7roie4MwbN6EMCziEK)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 7 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))$

Let $ty_2Epath_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epath_2Epath A0 A1) \tag{1}$$

Let $c_2Epath_2Elast : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epath_2Elast A_27a A_27b \in (A_27a (ty_2Epath_2Epath A_27a A_27b)) \tag{2}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{3}$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \quad (4)$$

Let $c_2Epath_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2EfromPath\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod\ A_27b\ A_27a)))^{(ty_2Epath_2Epath\ A_27a\ A_27b)}) \quad (5)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (6)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (8)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (9)$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 9 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (10)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (11)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (12)$$

Definition 11 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))\ c_2Enum_2E0)$

Definition 12 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (14)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in \\ (((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)}) \quad (15)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (16)$$

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (17)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (18)$$

Definition 14 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in \\ ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (19)$$

Definition 15 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Definition 16 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.))$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in \\ ((ty_2Ellist_2Ellist\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum}}) \quad (20)$$

Definition 18 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota.\lambda V0h \in A_27a.\lambda V1t \in (ty_2Ellist_2Ellist\ A_27a)$

Definition 19 We define c_Eone_Eone to be $(ap (c_Emin_E_40 ty_Eone_Eone) (\lambda V0x \in ty_Eone_Eone$

Definition 20 We define $c_Ebool_E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_E_3D_3D_3E V0t) c_Ebool_E_7E$

Definition 21 We define c_Esum_EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_Esum_EABS$

Definition 22 We define $c_EOption_ENONE$ to be $\lambda A_27a : \iota.(ap (c_EOption_EOption_ABS A_27a) (c$

Definition 23 We define c_Ellist_ELNIL to be $\lambda A_27a : \iota.(ap (c_Ellist_Ellist_abs A_27a) (\lambda V0n \in ty$

Definition 24 We define $c_Ellist_ELFINITE$ to be $\lambda A_27a : \iota.(\lambda V0a0 \in (ty_Ellist_Ellist A_27a).(ap (c$

Definition 25 We define $c_Epath_Efinite$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0sigma \in (ty_Epath_Epath A$

Let $c_Epair_E_EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_E_EFST \\ A_27a A_27b \in (A_27a^{(ty_Epair_Eprod A_27a A_27b)}) \end{aligned} \quad (21)$$

Definition 26 We define c_Epath_Efirst to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (ty_Epath_Epath A_27a A$

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_EABS_prod \\ A_27a A_27b \in ((ty_Epair_Eprod A_27a A_27b)^{(2^{A_27b} A_27a)}) \end{aligned} \quad (22)$$

Definition 27 We define $c_Epair_E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c$

Let $c_Epath_EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epath_EtoPath \\ A_27a A_27b \in ((ty_Epath_Epath A_27a A_27b)^{(ty_Epair_Eprod A_27a (ty_Ellist_Ellist (ty_Epair_Eprod$$
 \end{aligned} \quad (23)

Definition 28 We define $c_Epath_E_pcons$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1r \in A_27b.\lambda V2p$

Definition 29 We define $c_Epath_E_stopped_at$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.(ap (c_Epath$

Let $c_Epath_E_plink : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epath_E_plink \\ A_27a A_27b \in (((ty_Epath_Epath A_27a A_27b)^{(ty_Epath_Epath A_27a A_27b)})^{(ty_Epath_Epath A_27a A$$
 \end{aligned} \quad (24)

Assume the following.

$$True \quad (25)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p V0t)))))) \end{aligned} \quad (36)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (37)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (38)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0a \in A_{.27a}.(\exists V1x \in A_{.27a}.(V1x = V0a))) \quad (39)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (40)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27a}.(((ap (c_{.2Epath}._2Estopped_at A_{.27a} A_{.27b}) V0x) = (ap (c_{.2Epath}._2Estopped_at A_{.27a} A_{.27b}) V1y)) \Leftrightarrow (V0x = V1y)))) \quad (41)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1r \in A_{.27b}.(\forall V2p \in (ty_{.2Epath}._2Epath A_{.27a} A_{.27b}).(\forall V3y \in A_{.27a}.(\forall V4s \in A_{.27b}.(\forall V5q \in (ty_{.2Epath}._2Epath A_{.27a} A_{.27b}).(((ap (ap (ap (c_{.2Epath}._2Epcns A_{.27a} A_{.27b}) V0x) V1r) V2p) = (ap (ap (ap (c_{.2Epath}._2Epcns A_{.27a} A_{.27b}) V3y) V4s) V5q)) \Leftrightarrow ((V0x = V3y) \wedge ((V1r = V4s) \wedge (V2p = V5q)))))))))) \quad (42)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27a}.(\forall V2r \in A_{.27b}.(\forall V3p \in (ty_{.2Epath}._2Epath A_{.27a} A_{.27b}).((\neg((ap (c_{.2Epath}._2Estopped_at A_{.27a} A_{.27b}) V0x) = (ap (ap (ap (c_{.2Epath}._2Epcns A_{.27a} A_{.27b}) V1y) V2r) V3p)))) \wedge (\neg((ap (ap (ap (c_{.2Epath}._2Epcns A_{.27a} A_{.27b}) V1y) V2r) V3p) = (ap (c_{.2Epath}._2Estopped_at A_{.27a} A_{.27b}) V0x)))))))) \quad (43)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0x \in A_27a. ((ap\ (c_2Epath_2Elast\ A_27a\ A_27b)\ (ap\ (c_2Epath_2Estopped_at \\
& \quad A_27a\ A_27b)\ V0x)) = V0x)) \wedge (\forall V1x \in A_27a. (\forall V2r \in A_27b. \\
& \quad (\forall V3p \in (ty_2Epath_2Epath\ A_27a\ A_27b). ((ap\ (c_2Epath_2Elast \\
& \quad A_27a\ A_27b)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A_27a\ A_27b)\ V1x)\ V2r) \\
& \quad V3p)) = (ap\ (c_2Epath_2Elast\ A_27a\ A_27b)\ V3p)))))) \\
& \hspace{10em} (44)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0P \in (2^{(ty_2Epath_2Epath\ A_27a\ A_27b)}). ((\forall V1x \in \\
& \quad A_27a. (p\ (ap\ V0P\ (ap\ (c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V1x)))) \wedge \\
& \quad (\forall V2x \in A_27a. (\forall V3r \in A_27b. (\forall V4p \in (ty_2Epath_2Epath \\
& \quad A_27a\ A_27b). ((p\ (ap\ (c_2Epath_2Efinite\ A_27a\ A_27b)\ V4p)) \wedge (\\
& \quad p\ (ap\ V0P\ V4p)))) \Rightarrow (p\ (ap\ V0P\ (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A_27a\ A_27b) \\
& \quad V2x)\ V3r)\ V4p)))))) \Rightarrow (\forall V5q \in (ty_2Epath_2Epath\ A_27a\ A_27b). \\
& \quad ((p\ (ap\ (c_2Epath_2Efinite\ A_27a\ A_27b)\ V5q)) \Rightarrow (p\ (ap\ V0P\ V5q)))))) \\
& \hspace{10em} (45)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad (\forall V0x \in A_27a. (\forall V1p \in (ty_2Epath_2Epath\ A_27a\ A_27b). \\
& \quad ((ap\ (ap\ (c_2Epath_2Eplink\ A_27a\ A_27b)\ (ap\ (c_2Epath_2Estopped_at \\
& \quad A_27a\ A_27b)\ V0x))\ V1p) = V1p))) \wedge (\forall V2x \in A_27a. (\forall V3r \in \\
& \quad A_27b. (\forall V4p1 \in (ty_2Epath_2Epath\ A_27a\ A_27b). (\forall V5p2 \in \\
& \quad (ty_2Epath_2Epath\ A_27a\ A_27b). ((ap\ (ap\ (c_2Epath_2Eplink\ A_27a \\
& \quad A_27b)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A_27a\ A_27b)\ V2x)\ V3r)\ V4p1)) \\
& \quad V5p2) = (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A_27a\ A_27b)\ V2x)\ V3r)\ (ap\ (\\
& \quad ap\ (c_2Epath_2Eplink\ A_27a\ A_27b)\ V4p1)\ V5p2)))))) \\
& \hspace{10em} (46)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \hspace{10em} (47)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \hspace{10em} (48)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \hspace{10em} (49)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \hspace{10em} (50)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg(p\ V1q)))) \hspace{10em} (51)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0p \in (ty_2Epath_2Epath\ A_27a\ A_27b).((p\ (ap\ (c_2Epath_2Efinite \\ & A_27a\ A_27b)\ V0p)) \Rightarrow ((\exists V1x \in A_27a.(V0p = (ap\ (c_2Epath_2Estopped_at \\ & A_27a\ A_27b)\ V1x))) \vee (\exists V2p_27 \in (ty_2Epath_2Epath\ A_27a \\ & A_27b).(\exists V3l \in A_27b.(\exists V4s \in A_27a.(V0p = (ap\ (ap\ (\\ & c_2Epath_2Eplink\ A_27a\ A_27b)\ V2p_27)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcons \\ & A_27a\ A_27b)\ (ap\ (c_2Epath_2Elast\ A_27a\ A_27b)\ V2p_27))\ V3l)\ (ap \\ & (c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V4s)))))))))) \end{aligned}$$