

# thm\_2Epath\_2Efinites\_paths\_SN (TMR-CYLP3hx88QAPk3zLnemTFZux9kD3wZXj)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V0x))$

**Definition 3** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. (\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 4** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota. (ap (ap (c\_2Ecombin\_2ES A\_27a (A\_27a^{A\_27a}) A\_27b) A\_27c) A\_27d)$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (2)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (3)$$

**Definition 5** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 6** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})) A\_27b) A\_27c))$

**Definition 7** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2. V0t))$ .

**Definition 8** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p \ P \Rightarrow p \ Q)$  of type  $\iota$ .

**Definition 9** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF))$

**Definition 10** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. inj\_o (V0t1 \ P \Rightarrow p \ V2t))))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (5)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (6)$$

**Definition 11** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num m)$

**Definition 12** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\lambda x.x \in A \wedge p) \text{ of type } \iota \Rightarrow \iota.$

**Definition 13** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A) P)))$

**Definition 14** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(c\_2Ebool\_2E\_3F (c\_2Eprim\_rec\_2E\_3C m n))$

**Definition 15** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(c\_2Eprim\_rec\_2E\_3C (c\_2Earithmetic\_2E\_3E m n))$

**Definition 16** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (c\_2Ebool\_2E\_21 t1 t2))))$

**Definition 17** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(c\_2Ebool\_2E\_5C\_2F (c\_2Earithmetic\_2E\_3E m n))$

**Definition 18** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(c\_2Ebool\_2E\_5C\_2F (c\_2Earithmetic\_2E\_3C m n))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (7)$$

**Definition 19** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP).$

**Definition 20** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(c\_2Ebool\_2E\_21 t1 t2))))$

**Definition 21** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebool\_2E\_21 2) (c\_2Ebool\_2E\_21 m)))$

**Definition 22** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0.$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 23** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B n))$

**Definition 24** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B n))$

Let  $c\_2Earithmetic\_2EXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 25** We define  $c\_2Eenumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 26** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty } (ty\_2Epair\_2Eprod \\ A0 A1) \end{aligned} \quad (12)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$\text{nonempty } ty\_2Eone\_2Eone \quad (13)$$

**Definition 27** We define  $c\_2Eone\_2Eone$  to be  $(ap (c\_2Emin\_2E\_40 ty\_2Eone\_2Eone) (\lambda V0x \in ty\_2Eone\_2Eone))$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A0.\text{nonempty } A0 \Rightarrow \forall A1.\text{nonempty } A1 \Rightarrow \text{nonempty } (ty\_2Esum\_2Esum \\ A0 A1) \end{aligned} \quad (14)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow c\_2Esum\_2EABS\_sum \\ A\_27a A\_27b \in ((ty\_2Esum\_2Esum A\_27a A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \end{aligned} \quad (15)$$

**Definition 28** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27b. (ap (c\_2Esum\_2EABS$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (ty\_2Eoption\_2Eoption A0) \quad (16)$$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS A\_27a \in \\ ((ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Esum\_2Esum A\_27a ty\_2Eone\_2Eone)}) \end{aligned} \quad (17)$$

**Definition 29** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota.(ap (c\_2Eoption\_2Eoption\_ABS A\_27a) (A\_27a))$

Let  $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ellist\_2Ellist A0) \quad (18)$$

Let  $c\_2Ellist\_2Ellist\_abs : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ellist\_2Ellist\_abs A\_27a \in \\ & ((ty\_2Ellist\_2Ellist A\_27a)^{(ty\_2Eoption\_2Eoption A\_27a)^{ty\_2Enum\_2Enum}}) \end{aligned} \quad (19)$$

**Definition 30** We define  $c\_2Ellist\_2ELNIL$  to be  $\lambda A\_27a : \iota.(ap (c\_2Ellist\_2Ellist\_abs A\_27a) (\lambda V0n \in ty\_2Ellist\_2Ellist A\_27a)))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ & A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (20)$$

**Definition 31** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair A\_27a A\_27b) (V0x V1y)))$

Let  $ty\_2Epath\_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epath\_2Epath \\ & A0 A1) \end{aligned} \quad (21)$$

Let  $c\_2Epath\_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epath\_2EtoPath \\ & A\_27a A\_27b \in ((ty\_2Epath\_2Epath A\_27a A\_27b)^{(ty\_2Epair\_2Eprod A\_27a (ty\_2Ellist\_2Ellist (ty\_2Epair\_2Eprod \\ & A\_27b A\_27a))))}) \end{aligned} \quad (22)$$

**Definition 32** We define  $c\_2Epath\_2Estopped\_at$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.(ap (c\_2Epath A\_27a A\_27b) (V0x)))$

**Definition 33** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b)^{A\_27c}).\lambda V1g \in (A\_27c)^{A\_27b}(ap (c\_2Ecombin A\_27a A\_27b A\_27c) (f g)))$

Let  $c\_2Epath\_2Epgenerate : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epath\_2Epgenerate \\ & A\_27a A\_27b \in (((ty\_2Epath\_2Epath A\_27a A\_27b)^{(A\_27b)^{ty\_2Enum\_2Enum}}))^{(A\_27a)^{ty\_2Enum\_2Enum}} \end{aligned} \quad (23)$$

Let  $c\_2Epath\_2EffromPath : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epath\_2EffromPath \\ & A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a (ty\_2Ellist\_2Ellist (ty\_2Epair\_2Eprod \\ & A\_27b A\_27a))))^{(ty\_2Epath\_2Epath A\_27a A\_27b)}) \end{aligned} \quad (24)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2ESND \\ & A\_27a A\_27b \in (A\_27b)^{(ty\_2Epair\_2Eprod A\_27a A\_27b)} \end{aligned} \quad (25)$$

Let  $c\_2Ellist\_2Ellist\_rep : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A_{\_27a}.nonempty A_{\_27a} \Rightarrow c\_2Ellist\_2Ellist\_rep A_{\_27a} \in \\ & (((ty\_2Eoption\_2Eoption A_{\_27a})^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist A_{\_27a})}) \end{aligned} \quad (26)$$

**Definition 34** We define  $c\_2Esum\_2EINL$  to be  $\lambda A_{\_27a} : \iota. \lambda A_{\_27b} : \iota. \lambda V0e \in A_{\_27a}.(ap (c\_2Esum\_2EABS$

**Definition 35** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A_{\_27a} : \iota. \lambda V0x \in A_{\_27a}.(ap (c\_2Eoption\_2Eoption$

**Definition 36** We define  $c\_2Ellist\_2ELCONS$  to be  $\lambda A_{\_27a} : \iota. \lambda V0h \in A_{\_27a}. \lambda V1t \in (ty\_2Ellist\_2Ellist A_{\_27a})$

**Definition 37** We define  $c\_2Ellist\_2ELFINITE$  to be  $\lambda A_{\_27a} : \iota. (\lambda V0a0 \in (ty\_2Ellist\_2Ellist A_{\_27a}).(ap (c\_2Ellist\_2Ellist A_{\_27a}) V0a0))$

**Definition 38** We define  $c\_2Epath\_2Efinite$  to be  $\lambda A_{\_27a} : \iota. \lambda A_{\_27b} : \iota. \lambda V0sigma \in (ty\_2Epath\_2Epath A_{\_27a})$

**Definition 39** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A_{\_27a} : \iota. (\lambda V0x \in A_{\_27a}.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A_{\_27a}.nonempty A_{\_27a} \Rightarrow \forall A_{\_27b}.nonempty A_{\_27b} \Rightarrow c\_2Epair\_2EFST \\ & A_{\_27a} A_{\_27b} \in (A_{\_27a}^{(ty\_2Epair\_2Eprod A_{\_27a} A_{\_27b})}) \end{aligned} \quad (27)$$

**Definition 40** We define  $c\_2Epath\_2Efirst$  to be  $\lambda A_{\_27a} : \iota. \lambda A_{\_27b} : \iota. \lambda V0p \in (ty\_2Epath\_2Epath A_{\_27a} A_{\_27b})$

**Definition 41** We define  $c\_2Epath\_2Epcons$  to be  $\lambda A_{\_27a} : \iota. \lambda A_{\_27b} : \iota. \lambda V0x \in A_{\_27a}. \lambda V1r \in A_{\_27b}. \lambda V2p \in (ty\_2Epath\_2Epath A_{\_27b} A_{\_27a})$

**Definition 42** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A_{\_27a} : \iota. \lambda A_{\_27b} : \iota. \lambda A_{\_27c} : \iota. \lambda V0f \in ((A_{\_27c}^{A\_27a})^{(ty\_2Epair\_2Eprod A_{\_27a} A_{\_27c})})$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A_{\_27a}.nonempty A_{\_27a} \Rightarrow \forall A_{\_27b}.nonempty A_{\_27b} \Rightarrow c\_2Epred\_set\_2EGSPEC \\ & A_{\_27a} A_{\_27b} \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A_{\_27a} A_{\_27b})}) \end{aligned} \quad (28)$$

**Definition 43** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A_{\_27a} : \iota. (\lambda V0x \in A_{\_27a}. c\_2Ebool\_2ET).$

**Definition 44** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A_{\_27a} : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EGSPEC A_{\_27a} A_{\_27a}) V0s V1t)$

**Definition 45** We define  $c\_2Epath\_2Eokpath\_f$  to be  $\lambda A_{\_27a} : \iota. \lambda A_{\_27b} : \iota. \lambda V0R \in (((2^{A\_27a})^{A\_27b})^{A\_27a})$

**Definition 46** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A_{\_27a} : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}).(ap (c\_2Epred\_set\_2EGSPEC A_{\_27a} A_{\_27a}) V0s V1t)$

**Definition 47** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A_{\_27a} : \iota. \lambda V0P \in (2^{(2^{A\_27a})}).(ap (c\_2Epred\_set\_2EGSPEC A_{\_27a} A_{\_27a}) V0P)$

**Definition 48** We define  $c\_2EfixedPoint\_2Egfp$  to be  $\lambda A_{\_27a} : \iota. \lambda V0f \in ((2^{A\_27a})^{(2^{A\_27a})}).(ap (c\_2Epred\_set\_2EGSPEC A_{\_27a} A_{\_27a}) V0f)$

**Definition 49** We define  $c\_2Epath\_2Eokpath$  to be  $\lambda A_{\_27a} : \iota. \lambda A_{\_27b} : \iota. \lambda V0R \in (((2^{A\_27a})^{A\_27b})^{A\_27a})$

**Definition 50** We define  $c\_2Erelation\_2EWF$  to be  $\lambda A\_27a : \iota. \lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap (c\_2Ebool\_2E\_21$

**Definition 51** We define  $c\_2Epath\_2ESN$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0R \in (((2^{A\_27a})^{A\_27b})^{A\_27a}).(ap (c\_2Epath\_2ESN$

**Definition 52** We define  $c\_2Eprim\_rec\_2Ewellfounded$  to be  $\lambda A\_27a : \iota. \lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap c\_2Ebool\_2E\_21$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m)) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum. \\ & ((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap ( \\ & ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B \\ & (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\ & V0m) V1n))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\ & V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum. \\ & ((ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B \\ & V1n) V0m)))) \end{aligned} \quad (31)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap c\_2Enum\_2ESUC V0m) = (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \quad (32)$$

Assume the following.

$$True \quad (33)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (34)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a. (p V0t) \Leftrightarrow (p V0t)))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & \quad True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & \quad (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & \quad ((\neg False) \Leftrightarrow True)))) \end{aligned} \quad (39)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (40)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & \quad (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & \quad (p V0t))))))) \end{aligned} \quad (42)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in A\_27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p (ap V0P V2x))))))) \quad (43)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in A\_27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p (ap V0P V2x))))))) \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in \\ & \quad 2.((\forall V2x \in A\_27a.((p (ap V0P V2x)) \Rightarrow (p V1Q))) \Leftrightarrow ((\exists V3x \in A\_27a.(p (ap V0P V3x))) \Rightarrow (p V1Q))))) \end{aligned} \quad (45)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (47)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ & 2. (((((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27))))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}). (\forall V1v \in \\ & A\_27a. ((\forall V2x \in A\_27a. ((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p ( \\ & ap V0f V1v)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \forall A\_27c. \\ & \text{nonempty } A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27a^{A\_27c}). \\ & (\forall V2x \in A\_27c. ((ap (ap (ap (c\_2Ecombin\_2Eo A\_27c A\_27b A\_27a) \\ & V0f) V1g) V2x) = (ap V0f (ap V1g V2x))))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \forall A\_27c. \\ & \text{nonempty } A\_27c \Rightarrow \forall A\_27d.\text{nonempty } A\_27d \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). \\ & (\forall V1g \in (A\_27a^{A\_27c}). (\forall V2h \in (A\_27c^{A\_27d}). ((ap ( \\ & ap (c\_2Ecombin\_2Eo A\_27d A\_27b A\_27a) V0f) (ap (ap (c\_2Ecombin\_2Eo \\ & A\_27d A\_27a A\_27c) V1g) V2h)) = (ap (ap (c\_2Ecombin\_2Eo A\_27d A\_27b \\ & A\_27c) (ap (ap (c\_2Ecombin\_2Eo A\_27c A\_27b A\_27a) V0f) V1g)) V2h))))))) \end{aligned} \quad (52)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap (c\_2Ecombin\_2Eo \\ & A\_27a) V0x) = V0x)) \quad (53)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& (ap c\_2Earithmetic\_2E\_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& ((\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
(ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge (((ap c\_2Enum\_2ESUC \\
c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
c\_2Earithmetic\_2EZERO)))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. \\
& (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Enum\_2ESUC V17n)))) \wedge (((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
(ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Eprim\_rec\_2EPRE V18n)))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& ((\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V30m) V29n)))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
V32n)))) \wedge ((\forall V33n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
c\_2Enum\_2E0) V33n)) \Leftrightarrow False)) \wedge ((\forall V34n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
V34n)) \Leftrightarrow False)))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \\ & \forall V0x \in A_{27a}.(\forall V1r \in A_{27b}.(\forall V2p \in (ty\_2Epath\_2Epath \\ & A_{27a} A_{27b}).(\forall V3y \in A_{27a}.(\forall V4s \in A_{27b}.(\forall V5q \in \\ & (ty\_2Epath\_2Epath A_{27a} A_{27b}).(((ap (ap (ap (c\_2Epath\_2Epcos \\ & A_{27a} A_{27b}) V0x) V1r) V2p) = (ap (ap (ap (c\_2Epath\_2Epcos A_{27a} \\ & A_{27b}) V3y) V4s) V5q)) \Leftrightarrow ((V0x = V3y) \wedge ((V1r = V4s) \wedge (V2p = V5q))))))))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \\ & (\forall V0x \in A_{27a}.((ap (c\_2Epath\_2Efirst A_{27a} A_{27b}) (ap (c\_2Epath\_2Estopped\_at \\ & A_{27a} A_{27b}) V0x)) = V0x)) \wedge (\forall V1x \in A_{27a}.(\forall V2r \in A_{27b}. \\ & (\forall V3p \in (ty\_2Epath\_2Epath A_{27a} A_{27b}).((ap (c\_2Epath\_2Efirst \\ & A_{27a} A_{27b}) (ap (ap (c\_2Epath\_2Epcos A_{27a} A_{27b}) V1x) V2r) \\ & V3p)) = V1x)))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \\ & \forall V0f \in (A_{27a}^{ty\_2Enum\_2Enum}).(\forall V1g \in (A_{27b}^{ty\_2Enum\_2Enum}). \\ & ((ap (ap (c\_2Epath\_2Epgenerate A_{27a} A_{27b}) V0f) V1g) = (ap (ap ( \\ & ap (c\_2Epath\_2Epcos A_{27a} A_{27b}) (ap V0f c\_2Enum\_2E0)) (ap V1g \\ & c\_2Enum\_2E0)) (ap (ap (c\_2Epath\_2Epgenerate A_{27a} A_{27b}) (ap ( \\ & ap (c\_2Ecombin\_2Eo ty\_2Enum\_2Enum A_{27a} ty\_2Enum\_2Enum) V0f) \\ & c\_2Enum\_2ESUC)) (ap (ap (c\_2Ecombin\_2Eo ty\_2Enum\_2Enum A_{27b} \\ & ty\_2Enum\_2Enum) V1g) c\_2Enum\_2ESUC))))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \\ & \forall V0f \in (A_{27a}^{ty\_2Enum\_2Enum}).(\forall V1g \in (A_{27b}^{ty\_2Enum\_2Enum}). \\ & (\neg(p (ap (c\_2Epath\_2Efinite A_{27a} A_{27b}) (ap (ap (c\_2Epath\_2Epgenerate \\ & A_{27a} A_{27b}) V0f) V1g)))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \\ & \forall V0R \in (((2^{A_{27a}})^{A_{27b}})^{A_{27a}}).(\forall V1P \in (2^{(ty\_2Epath\_2Epath A_{27a} A_{27b})}). \\ & ((\forall V2x \in A_{27a}.(\forall V3r \in A_{27b}.(\forall V4p \in (ty\_2Epath\_2Epath \\ & A_{27a} A_{27b}).((p (ap V1P (ap (ap (c\_2Epath\_2Epcos A_{27a} A_{27b}) \\ & V2x) V3r) V4p)) \Rightarrow ((p (ap (ap (ap V0R V2x) V3r) (ap (c\_2Epath\_2Efirst \\ & A_{27a} A_{27b}) V4p)) \wedge (p (ap V1P V4p))))))) \Rightarrow (\forall V5p \in (ty\_2Epath\_2Epath \\ & A_{27a} A_{27b}).((p (ap V1P V5p)) \Rightarrow (p (ap (ap (c\_2Epath\_2Eokpath A_{27a} \\ & A_{27b}) V0R) V5p))))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\ ((p (ap (c_2Erelation_2EWF A_27a) V0R)) \Leftrightarrow (p (ap (c_2Eprim_rec_2Ewellfounded \\ A_27a) V0R)))) \end{aligned} \quad (60)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (61)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (62)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\ ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\ ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (64)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (65)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\ (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg( \\ p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\ (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\ (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (( \\ (\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (68)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (69)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\ & \forall V0R \in (((2^{A\_27a})^{A\_27b})^{A\_27a}).((\forall V1p \in (\text{ty\_2Epath\_2Epath} \\ & A\_27a A\_27b).((p (\text{ap} (\text{ap} (\text{c\_2Epath\_2Eokpath} A\_27a A\_27b) V0R) V1p))) \Rightarrow \\ & (p (\text{ap} (\text{c\_2Epath\_2Efinite} A\_27a A\_27b) V1p)))) \Rightarrow (p (\text{ap} (\text{c\_2Epath\_2ESN} \\ & A\_27a A\_27b) V0R))) \end{aligned}$$