

thm_2Epath_2Efinite_pconcat (TMD- jpWw8mji3XwNXTo55ypJkYKMGQGsWZTx)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \tag{2}$$

Let $ty_2Epath_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epath_2Epath A0 A1) \tag{3}$$

Let $c_2Epath_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epath_2EfromPath A.27a A.27b \in ((ty_2Epair_2Eprod A.27a (ty_2Ellist_2Ellist (ty_2Epair_2Eprod A.27b A.27a))) (ty_2Epath_2Epath A.27a A.27b)) \tag{4}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2ESND A.27a A.27b \in (A.27b (ty_2Epair_2Eprod A.27a A.27b)) \tag{5}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{6}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \tag{7}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (8)$$

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 4 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \quad (10)$$

Definition 5 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ P)\ V0P))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ (c_2Enum_2ESUC_REP\ m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n))$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (12)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (13)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)} \quad (14)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (15)$$

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ V1t2)\ V0t1))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (16)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (17)$$

Definition 11 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (18)$$

Definition 12 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Definition 13 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2. V0t))$.

Definition 14 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P\ x) \text{ then } (the\ (\lambda x. x \in A \wedge P\ x)\ V0t)$ of type $\iota \Rightarrow \iota$.

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap\ (c_2Emin_2E_40\ A_27a)\ V2t2)\ V1t1))\ V0t)$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in ((ty_2Ellist_2Ellist\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum}}) \quad (19)$$

Definition 16 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist\ A_27a)\ V0h. (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ V1t)$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a)\ V0P)))$

Definition 18 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone. V0x))$

Definition 19 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_3F)\ V0t)$

Definition 20 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 21 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0t)$

Definition 22 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ (\lambda V0n \in ty_2Ellist_2Ellist\ A_27a. V0n))$

Definition 23 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ V1t2)\ V0t1))$

Definition 24 We define $c_2Ellist_2ELFINITE$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ V0a0))$

Definition 25 We define $c_2Epath_2Efinite$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0sigma \in (ty_2Epath_2Epath\ A_27a\ A_27b). (ap\ (c_2Emin_2E_40\ A_27a)\ V0sigma)$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ & A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (20)$$

Definition 26 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $c_2Epath_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2EtoPath \\ & A_27a\ A_27b \in ((ty_2Epath_2Epath\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod\ A_27a\ A_27b))})} \end{aligned} \quad (21)$$

Definition 27 We define $c_2Epath_2Estopped_at$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.(ap\ (c_2Epath$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ & A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (22)$$

Definition 28 We define $c_2Epath_2Efirst$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (ty_2Epath_2Epath\ A_27a\ A$

Definition 29 We define $c_2Epath_2Eacons$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1r \in A_27b.\lambda V2p$

Let $c_2Ellist_2ELAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2ELAPPEND\ A_27a \in (((\\ & ty_2Ellist_2Ellist\ A_27a)^{(ty_2Ellist_2Ellist\ A_27a)}(ty_2Ellist_2Ellist\ A_27a)) \end{aligned} \quad (23)$$

Definition 30 We define $c_2Epath_2Econcat$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p1 \in (ty_2Epath_2Epath\ A_27a\ A$

Assume the following.

$$True \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\ & (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & p\ V0t)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).((\exists V2x \in A_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \wedge (\exists V3x \in A_27a.(p\ (ap\ V1Q\ V3x))))))) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a.(p\ (ap\ V1Q\ V3x))))))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \quad (43)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (44)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (45)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow ((\forall V0x \in A_27a.((p\ (c_2Epath_2Efinite\ A_27a\ A_27b)\ (ap\ (c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V0x)) \Leftrightarrow True)) \wedge (\forall V1x \in A_27a.(\forall V2r \in A_27b.(\forall V3p \in (ty_2Epath_2Epath\ A_27a\ A_27b).((p\ (ap\ (c_2Epath_2Efinite\ A_27a\ A_27b)\ (ap\ (ap\ (ap\ (c_2Epath_2Econs\ A_27a\ A_27b)\ V1x)\ V2r)\ V3p))) \Leftrightarrow (p\ (ap\ (c_2Epath_2Efinite\ A_27a\ A_27b)\ V3p)))))) \quad (46)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0P \in (2^{(ty_2Epath_2Epath\ A.27a\ A.27b)}).((\forall V1x \in \\
& A.27a.(p\ (ap\ V0P\ (ap\ (c_2Epath_2Estopped_at\ A.27a\ A.27b)\ V1x)))) \wedge \\
& \quad (\forall V2x \in A.27a.(\forall V3r \in A.27b.(\forall V4p \in (ty_2Epath_2Epath \\
& \quad A.27a\ A.27b).(((p\ (ap\ (c_2Epath_2Efinite\ A.27a\ A.27b)\ V4p)) \wedge (\\
& p\ (ap\ V0P\ V4p)))) \Rightarrow (p\ (ap\ V0P\ (ap\ (ap\ (ap\ (c_2Epath_2Epcns\ A.27a\ A.27b) \\
& \quad V2x)\ V3r)\ V4p)))))) \Rightarrow (\forall V5q \in (ty_2Epath_2Epath\ A.27a\ A.27b). \\
& \quad ((p\ (ap\ (c_2Epath_2Efinite\ A.27a\ A.27b)\ V5q)) \Rightarrow (p\ (ap\ V0P\ V5q)))))) \\
& \hspace{15em} (47)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow ((\forall V0x \in A.27a. \\
& \quad (\forall V1lab \in A.27b.(\forall V2p2 \in (ty_2Epath_2Epath\ A.27a \\
& A.27b).((ap\ (ap\ (ap\ (c_2Epath_2Epcncat\ A.27a\ A.27b)\ (ap\ (c_2Epath_2Estopped_at \\
& \quad A.27a\ A.27b)\ V0x))\ V1lab)\ V2p2) = (ap\ (ap\ (ap\ (c_2Epath_2Epcns\ A.27a \\
& \quad A.27b)\ V0x)\ V1lab)\ V2p2)))) \wedge (\forall V3x \in A.27c.(\forall V4r \in \\
& \quad A.27d.(\forall V5p \in (ty_2Epath_2Epath\ A.27c\ A.27d).(\forall V6lab \in \\
& \quad A.27d.(\forall V7p2 \in (ty_2Epath_2Epath\ A.27c\ A.27d).((ap\ (ap \\
& \quad (ap\ (c_2Epath_2Epcncat\ A.27c\ A.27d)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcns \\
& \quad A.27c\ A.27d)\ V3x)\ V4r)\ V5p))\ V6lab)\ V7p2) = (ap\ (ap\ (ap\ (c_2Epath_2Epcns \\
& \quad A.27c\ A.27d)\ V3x)\ V4r)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcncat\ A.27c\ A.27d) \\
& \quad V5p)\ V6lab)\ V7p2)))))))))) \\
& \hspace{15em} (48)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0p1 \in (ty_2Epath_2Epath\ A.27a\ A.27b).(\forall V1lab \in \\
& \quad A.27b.(\forall V2p2 \in (ty_2Epath_2Epath\ A.27a\ A.27b).(\forall V3x \in \\
& A.27a.((\neg((ap\ (ap\ (ap\ (c_2Epath_2Epcncat\ A.27a\ A.27b)\ V0p1)\ V1lab) \\
& \quad V2p2) = (ap\ (c_2Epath_2Estopped_at\ A.27a\ A.27b)\ V3x)))) \wedge (\neg((ap \\
& (c_2Epath_2Estopped_at\ A.27a\ A.27b)\ V3x) = (ap\ (ap\ (ap\ (c_2Epath_2Epcncat \\
& \quad A.27a\ A.27b)\ V0p1)\ V1lab)\ V2p2)))))) \\
& \hspace{15em} (49)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0x \in A_27a. (\forall V1r \in A_27b. (\forall V2p \in (ty_2Epath_2Epath \\
& \quad A_27a\ A_27b). (\forall V3p1 \in (ty_2Epath_2Epath\ A_27a\ A_27b). (\\
& \quad \forall V4lab \in A_27b. (\forall V5p2 \in (ty_2Epath_2Epath\ A_27a\ A_27b). \\
& \quad (((ap\ (ap\ (ap\ (c_2Epath_2Epathconcat\ A_27a\ A_27b)\ V3p1)\ V4lab)\ V5p2) = \\
& \quad (ap\ (ap\ (ap\ (c_2Epath_2Epathcons\ A_27a\ A_27b)\ V0x)\ V1r)\ V2p)) \Leftrightarrow ((V4lab = \\
& \quad V1r) \wedge ((V3p1 = (ap\ (c_2Epath_2Epathstopped_at\ A_27a\ A_27b)\ V0x)) \wedge \\
& \quad (V2p = V5p2))) \vee (\exists V6p1_27 \in (ty_2Epath_2Epath\ A_27a\ A_27b). \\
& \quad ((V3p1 = (ap\ (ap\ (ap\ (c_2Epath_2Epathcons\ A_27a\ A_27b)\ V0x)\ V1r)\ V6p1_27)) \wedge \\
& \quad (V2p = (ap\ (ap\ (ap\ (c_2Epath_2Epathconcat\ A_27a\ A_27b)\ V6p1_27)\ V4lab) \\
& \quad V5p2)))))) \wedge (((ap\ (ap\ (ap\ (c_2Epath_2Epathcons\ A_27a\ A_27b)\ V0x)\ V1r) \\
& \quad V2p) = (ap\ (ap\ (ap\ (c_2Epath_2Epathconcat\ A_27a\ A_27b)\ V3p1)\ V4lab) \\
& \quad V5p2)) \Leftrightarrow ((V4lab = V1r) \wedge ((V3p1 = (ap\ (c_2Epath_2Epathstopped_at\ A_27a \\
& \quad A_27b)\ V0x)) \wedge (V2p = V5p2))) \vee (\exists V7p1_27 \in (ty_2Epath_2Epath \\
& \quad A_27a\ A_27b). ((V3p1 = (ap\ (ap\ (ap\ (c_2Epath_2Epathcons\ A_27a\ A_27b) \\
& \quad V0x)\ V1r)\ V7p1_27)) \wedge (V2p = (ap\ (ap\ (ap\ (c_2Epath_2Epathconcat\ A_27a \\
& \quad A_27b)\ V7p1_27)\ V4lab)\ V5p2))))))))) \\
& \hspace{15em} (50)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (52)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\
& \hspace{15em} (53)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\
& \hspace{15em} (54)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (55)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))) \\
& \hspace{15em} (56)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge ((p \ V0p) \vee (\neg(p \ V2r)))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{60}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\
& \forall V0p1 \in (ty_2Epath_2Epath \ A_27a \ A_27b). (\forall V1lab \in \\
& A_27b. (\forall V2p2 \in (ty_2Epath_2Epath \ A_27a \ A_27b). ((p \ (ap \ (\\
& c_2Epath_2Efinite \ A_27a \ A_27b) \ (ap \ (ap \ (ap \ (c_2Epath_2Econcat \\
& A_27a \ A_27b) \ V0p1) \ V1lab) \ V2p2))) \Leftrightarrow ((p \ (ap \ (c_2Epath_2Efinite \ A_27a \\
& A_27b) \ V0p1)) \wedge (p \ (ap \ (c_2Epath_2Efinite \ A_27a \ A_27b) \ V2p2))))))
\end{aligned}$$