

thm_2Epath_2Efirst_label_drop
(TMVmja7r25WtkY7fW2vSxpwpxnKvg3o2dD)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $ty_2Epath_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epath_2Epath A0 A1) \tag{1}$$

Let $c_2Epath_2Efirst_label : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epath_2Efirst_label A_27a A_27b \in (A_27b^{(ty_2Epath_2Epath A_27a A_27b)}) \tag{2}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \tag{3}$$

Let $c_2Epath_2Ent_label : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epath_2Ent_label A_27a A_27b \in ((A_27a^{(ty_2Epath_2Epath A_27b A_27a)})^{ty_2Enum_2Enum}) \tag{4}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{5}$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \quad (6)$$

Let $c_2Epath_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2EfromPath \\ & A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod \\ & \quad A_27b\ A_27a)))^{(ty_2Epath_2Epath\ A_27a\ A_27b)}) \end{aligned} \quad (7)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ & A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (8)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ & A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (9)$$

Definition 6 We define $c_2Epath_2Efirst$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0p \in (ty_2Epath_2Epath\ A_27a\ A_27b).$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. \dots))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ & A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{((A_27b)^{A_27a})}) \end{aligned} \quad (10)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2E_2C\ V0x\ V1y))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (11)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (12)$$

Definition 9 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 10 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (14)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (15)$$

Definition 12 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 13 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (16)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (17)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)} \quad (18)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (19)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (20)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (21)$$

Definition 14 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone}) \quad (22)$$

Definition 15 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption$

Definition 16 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{37}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg (p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\
& p \ V0t))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\
& 2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))))) \Rightarrow \\
& (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1a \in A_27a.((\exists V2x \in A_27a.((V2x = V1a) \wedge (p \ (ap \ V0P \ V2x)))) \Leftrightarrow (p \ (ap \ V0P \ V1a)))))) \tag{42}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\neg((ap \ c_2Enum_2ESUC \ V0n) = c_2Enum_2E0))) \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}).(((p (ap V0P c_2Enum_2E0)) \wedge \\
& (\forall V1n \in ty_2Enum_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c_2Enum_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum.(p (ap V0P V2n))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0P \in (2^{(ty_2Epath_2Epath A_27a A_27b)}).((\forall V1p \in \\
& (ty_2Epath_2Epath A_27a A_27b).(p (ap V0P V1p))) \Leftrightarrow ((\forall V2x \in \\
& A_27a.(p (ap V0P (ap (c_2Epath_2Estopped_at A_27a A_27b) V2x)))) \wedge \\
& (\forall V3x \in A_27a.(\forall V4r \in A_27b.(\forall V5p \in (ty_2Epath_2Epath \\
& A_27a A_27b).(p (ap V0P (ap (ap (ap (c_2Epath_2Epcns A_27a A_27b) \\
& V3x) V4r) V5p))))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0x \in A_27a.(\forall V1r \in A_27b.(\forall V2p \in (ty_2Epath_2Epath \\
& A_27a A_27b).((ap (c_2Epath_2Etail A_27a A_27b) (ap (ap (ap (c_2Epath_2Epcns \\
& A_27a A_27b) V0x) V1r) V2p)) = V2p))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& (\forall V0p \in (ty_2Epath_2Epath A_27b A_27a).((ap (ap (c_2Epath_2Epath_label \\
& A_27a A_27b) c_2Enum_2E0) V0p) = (ap (c_2Epath_2Efirst_label \\
& A_27b A_27a) V0p))) \wedge (\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in \\
& (ty_2Epath_2Epath A_27b A_27a).((ap (ap (c_2Epath_2Epath_label \\
& A_27a A_27b) (ap c_2Enum_2ESUC V1n)) V2p) = (ap (ap (c_2Epath_2Epath_label \\
& A_27a A_27b) V1n) (ap (c_2Epath_2Etail A_27b A_27a) V2p))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& (\forall V0x \in A_27a.((ap (c_2Epath_2EPL A_27a A_27b) (ap (c_2Epath_2Estopped_at \\
& A_27a A_27b) V0x)) = (ap (ap (c_2Epred_set_2EINSERT ty_2Enum_2Enum) \\
& c_2Enum_2E0) (c_2Epred_set_2EEMPTY ty_2Enum_2Enum)))) \wedge (\forall V1x \in \\
& A_27a.(\forall V2r \in A_27b.(\forall V3q \in (ty_2Epath_2Epath A_27a \\
& A_27b).((ap (c_2Epath_2EPL A_27a A_27b) (ap (ap (ap (c_2Epath_2Epcns \\
& A_27a A_27b) V1x) V2r) V3q)) = (ap (ap (c_2Epred_set_2EINSERT ty_2Enum_2Enum) \\
& c_2Enum_2E0) (ap (ap (c_2Epred_set_2EIMAGE ty_2Enum_2Enum ty_2Enum_2Enum) \\
& c_2Enum_2ESUC) (ap (c_2Epath_2EPL A_27a A_27b) V3q))))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0p \in (ty_2Epath_2Epath\ A.27a\ A.27b).(p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & ty_2Enum_2Enum)\ c_2Enum_2E0)\ (ap\ (c_2Epath_2EPL\ A.27a\ A.27b) \\ & V0p)))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & (\forall V0p \in (ty_2Epath_2Epath\ A.27a\ A.27b).(ap\ (ap\ (c_2Epath_2Edrop \\ & A.27a\ A.27b)\ c_2Enum_2E0)\ V0p) = V0p) \wedge (\forall V1n \in ty_2Enum_2Enum. \\ & (\forall V2p \in (ty_2Epath_2Epath\ A.27a\ A.27b).(ap\ (ap\ (c_2Epath_2Edrop \\ & A.27a\ A.27b)\ (ap\ c_2Enum_2ESUC\ V1n))\ V2p) = (ap\ (ap\ (c_2Epath_2Edrop \\ & A.27a\ A.27b)\ V1n)\ (ap\ (c_2Epath_2Etail\ A.27a\ A.27b)\ V2p)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A.27a)))))) \quad (51)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\ & A.27a.(\forall V2s \in (2^{A.27a}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a) \\ & V0x)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A.27a)\ V1y)\ V2s)))) \Leftrightarrow ((V0x = \\ & V1y) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V0x)\ V2s)))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0y \in A.27b.(\forall V1s \in (2^{A.27a}).(\forall V2f \in (A.27b^{A.27a}). \\ & ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & A.27a\ A.27b)\ V2f)\ V1s)))) \Leftrightarrow (\exists V3x \in A.27a.((V0y = (ap\ V2f\ V3x)) \wedge \\ & (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V3x)\ V1s)))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & ((ap\ c_2Enum_2ESUC\ V0m) = (ap\ c_2Enum_2ESUC\ V1n)) \Leftrightarrow (V0m = V1n)))) \end{aligned} \quad (54)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0i \in ty_2Enum_2Enum.(\forall V1p \in (ty_2Epath_2Epath \\ & A.27a\ A.27b).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Enum_2Enum)\ V0i)\ (\\ & ap\ (c_2Epath_2EPL\ A.27a\ A.27b)\ V1p)))) \Rightarrow ((ap\ (c_2Epath_2Efirst_label \\ & A.27a\ A.27b)\ (ap\ (ap\ (c_2Epath_2Edrop\ A.27a\ A.27b)\ V0i)\ V1p)) = (\\ & ap\ (ap\ (c_2Epath_2Entail_label\ A.27b\ A.27a)\ V0i)\ V1p)))) \end{aligned}$$