

thm_2Epath_2Efirst__pmap (TMHV s1zk2uqRXzBZnZcCx6zfwux8jzuRA3y)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x)$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \tag{2}$$

Let $ty_2Epath_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epath_2Epath A0 A1) \tag{3}$$

Let $c_2Epath_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epath_2EfromPath A.27a A.27b \in ((ty_2Epair_2Eprod A.27a (ty_2Ellist_2Ellist (ty_2Epair_2Eprod A.27b A.27a))) (ty_2Epath_2Epath A.27a A.27b)) \tag{4}$$

Let $c_2Epair_2E_EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2E_EFST A.27a A.27b \in (A.27a (ty_2Epair_2Eprod A.27a A.27b)) \tag{5}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota. (\lambda V 0P \in (2^{A.27a}). (ap (ap (c_2Emin_2E_3D (2^{A.27a}))) (\lambda V 1x \in 2.V 1x)))$

Definition 4 We define $c_2Epath_2Efirst$ to be $\lambda A.27a : \iota. \lambda A.27b : \iota. \lambda V 0p \in (ty_2Epath_2Epath A.27a A.27b)$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (6)$$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (7)$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (10)$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 9 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega)^{ty_2Enum_2Enum} \quad (11)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega)^{\omega} \quad (12)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (13)$$

Definition 11 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 12 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (14)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (15)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in \\ ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)} \quad (16)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (17)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (18)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in \\ ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (19)$$

Definition 13 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ e)$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in \\ ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone}) \quad (20)$$

Definition 14 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ x)$

Definition 15 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 16 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A)\ P)$ of type $\iota \Rightarrow \iota$.

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ (c_2Ebool_2E_21\ 2)\ t1\ t2))))$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in \\ ((ty_2Ellist_2Ellist\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum}}) \quad (21)$$

Definition 18 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota.\lambda V0h \in A_27a.\lambda V1t \in (ty_2Ellist_2Ellist A_27a A_27b) \rightarrow$
Let $c_2Epath_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epath_2EtoPath \\ A_27a A_27b \in & ((ty_2Epath_2Epath A_27a A_27b)^{(ty_2Epair_2Eprod A_27a (ty_2Ellist_2Ellist (ty_2Epair_2Eprod A_27a A_27b))})} \\ & (22) \end{aligned}$$

Definition 19 We define $c_2Epath_2Epcons$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1r \in A_27b.\lambda V2p \in$

Definition 20 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone))$

Definition 21 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 22 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS A_27a A_27b) V0e)$

Definition 23 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap (c_2Eoption_2Eoption_ABS A_27a) (c_2ENONE))$

Definition 24 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota.(ap (c_2Ellist_2Ellist_abs A_27a) (\lambda V0n \in ty_2Ellist_2Ellist A_27a) V0n))$

Definition 25 We define $c_2Epath_2Estopped_at$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.(ap (c_2Epath_2Estopped_at A_27a A_27b) V0x)$

Definition 26 We define $c_2Epair_2E_23_23$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f \in (A_27c A_27d) \rightarrow$

Let $c_2Ellist_2ELMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Ellist_2ELMAP \\ A_27a A_27b \in & (((ty_2Ellist_2Ellist A_27b)^{(ty_2Ellist_2Ellist A_27a)}^{(A_27b^{A_27a})}) \\ & (23) \end{aligned}$$

Definition 27 We define c_2Epath_2Emap to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f \in (A_27c A_27d) \rightarrow$

Assume the following.

$$True \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\ & True)) \quad (25) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \forall V0P \in (2^{(ty_2Epath_2Epath A_27a A_27b)}).((\forall V1p \in \\ & (ty_2Epath_2Epath A_27a A_27b).(p (ap V0P V1p))) \Leftrightarrow ((\forall V2x \in \\ & A_27a.(p (ap V0P (ap (c_2Epath_2Estopped_at A_27a A_27b) V2x)))) \wedge \\ & (\forall V3x \in A_27a.(\forall V4r \in A_27b.(\forall V5p \in (ty_2Epath_2Epath \\ & A_27a A_27b).(p (ap V0P (ap (ap (ap (c_2Epath_2Epcons A_27a A_27b) \\ & V3x) V4r) V5p)))))))))) \quad (26) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0x \in A_27a.((ap\ (c_2Epath_2Efirst\ A_27a\ A_27b)\ (ap\ (c_2Epath_2Estopped_at \\
& \quad A_27a\ A_27b)\ V0x)) = V0x)) \wedge (\forall V1x \in A_27a.(\forall V2r \in A_27b. \\
& \quad (\forall V3p \in (ty_2Epath_2Epath\ A_27a\ A_27b).((ap\ (c_2Epath_2Efirst \\
& \quad A_27a\ A_27b)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A_27a\ A_27b)\ V1x)\ V2r) \\
& \quad \quad V3p)) = V1x)))))) \\
& \hspace{15em} (27)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0f \in (A_27b^{A_27a}). \\
& \quad (\forall V1g \in (A_27c^{A_27d}).((\forall V2x \in A_27a.((ap\ (ap\ (ap\ (\\
& c_2Epath_2Epmap\ A_27a\ A_27d\ A_27b\ A_27c)\ V0f)\ V1g)\ (ap\ (c_2Epath_2Estopped_at \\
& \quad A_27a\ A_27d)\ V2x)) = (ap\ (c_2Epath_2Estopped_at\ A_27b\ A_27c)\ (\\
& \quad \quad ap\ V0f\ V2x)))))) \wedge (\forall V3x \in A_27a.(\forall V4r \in A_27d.(\forall V5p \in \\
& \quad (ty_2Epath_2Epath\ A_27a\ A_27d).((ap\ (ap\ (ap\ (c_2Epath_2Epmap \\
& \quad A_27a\ A_27d\ A_27b\ A_27c)\ V0f)\ V1g)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcons \\
& \quad A_27a\ A_27d)\ V3x)\ V4r)\ V5p)) = (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A_27b \\
& \quad A_27c)\ (ap\ V0f\ V3x))\ (ap\ V1g\ V4r))\ (ap\ (ap\ (ap\ (c_2Epath_2Epmap\ A_27a \\
& \quad \quad A_27d\ A_27b\ A_27c)\ V0f)\ V1g)\ V5p)))))))))) \\
& \hspace{15em} (28)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0f \in (A_27c^{A_27a}). \\
& \quad (\forall V1g \in (A_27d^{A_27b}).(\forall V2p \in (ty_2Epath_2Epath\ A_27a \\
& \quad A_27b).((ap\ (c_2Epath_2Efirst\ A_27c\ A_27d)\ (ap\ (ap\ (ap\ (c_2Epath_2Epmap \\
& \quad A_27a\ A_27b\ A_27c\ A_27d)\ V0f)\ V1g)\ V2p)) = (ap\ V0f\ (ap\ (c_2Epath_2Efirst \\
& \quad \quad A_27a\ A_27b)\ V2p)))))) \\
& \hspace{15em}
\end{aligned}$$