

thm_2Epath_2Eis_stopped_thm (TMWm-coEHcYseaME1jnXsQfNKfcgyStRMuEF)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \tag{2}$$

Let $ty_2Epath_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epath_2Epath A0 A1) \tag{3}$$

Let $c_2Epath_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epath_2EfromPath A_27a A_27b \in ((ty_2Epair_2Eprod A_27a (ty_2Ellist_2Ellist (ty_2Epair_2Eprod A_27b A_27a)))^{(ty_2Epath_2Epath A_27a A_27b)}) \tag{4}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \tag{5}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (6)$$

Definition 6 We define $c_2Epath_2Efirst$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0p \in (ty_2Epath_2Epath\ A_27a\ A_27b)$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. \dots))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (7)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2EABS_prod\ V0x\ V1y))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (10)$$

Definition 9 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 10 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (12)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ V0m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Definition 12 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n))$

Definition 13 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (15)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in \\ (((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)}) \quad (16)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (17)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (18)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (19)$$

Definition 14 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in \\ ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (20)$$

Definition 15 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Definition 16 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P\ x) \text{ then } (the\ (\lambda x. x \in A \wedge P\ x)) \text{ of type } \iota \Rightarrow \iota.$

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (V0t\ t1\ t2))))$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in \\ ((ty_2Ellist_2Ellist\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum}}) \quad (21)$$

Definition 18 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist\ A_27a)$

Let $c_2Epath_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2EtoPath \\ & A_27a\ A_27b \in ((ty_2Epath_2Epath\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod\ } \\ & \hspace{15em} (22) \end{aligned}$$

Definition 19 We define c_2Epath_2Epcns to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1r \in A_27b.\lambda V2p$

Definition 20 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E$

Definition 21 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone$

Definition 22 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS$

Definition 23 We define $c_2Eoption_2E_NONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_2E_ABS\ A_27a)\ (c_2Eoption_2E_NONE$

Definition 24 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota.(ap\ (c_2Ellist_2Ellist_2E_abs\ A_27a)\ (\lambda V0n \in ty_2Ellist_2ELNIL$

Definition 25 We define $c_2Epath_2Estopped_at$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.(ap\ (c_2Epath_2Estopped_at$

Definition 26 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 27 We define $c_2Epath_2Eis_stopped$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (ty_2Epath_2Epath\ A_27a\ A_27b$

Assume the following.

$$True \tag{23}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & \hspace{1.5em} V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{25}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in \\ & \hspace{1.5em} A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & \hspace{1.5em} ((\neg False) \Leftrightarrow True))) \end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ & \hspace{1.5em} A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p \ V0t))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0a \in A_27a. (\exists V1x \in A_27a. (V1x = V0a))) \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\
& \forall V0x \in A_27a. (\forall V1y \in A_27a. (((ap \ (c_2Epath_2Estopped_at \\
& A_27a \ A_27b) \ V0x) = (ap \ (c_2Epath_2Estopped_at \ A_27a \ A_27b) \ V1y)) \Leftrightarrow \\
& (V0x = V1y))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\
& \forall V0x \in A_27a. (\forall V1y \in A_27a. (\forall V2r \in A_27b. (\forall V3p \in \\
& (ty_2Epath_2Epath \ A_27a \ A_27b). (\neg((ap \ (c_2Epath_2Estopped_at \\
& A_27a \ A_27b) \ V0x) = (ap \ (ap \ (ap \ (c_2Epath_2Epcons \ A_27a \ A_27b) \ V1y) \\
& V2r) \ V3p)))) \wedge (\neg((ap \ (ap \ (ap \ (c_2Epath_2Epcons \ A_27a \ A_27b) \ V1y) \\
& V2r) \ V3p) = (ap \ (c_2Epath_2Estopped_at \ A_27a \ A_27b) \ V0x))))))
\end{aligned} \tag{32}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow \forall A_27c. \\
& nonempty \ A_27c \Rightarrow \forall A_27d.nonempty \ A_27d \Rightarrow ((\forall V0x \in A_27a. \\
& ((p \ (ap \ (c_2Epath_2Eis_stopped \ A_27a \ A_27b) \ (ap \ (c_2Epath_2Estopped_at \\
& A_27a \ A_27b) \ V0x))) \Leftrightarrow True)) \wedge (\forall V1x \in A_27c. (\forall V2r \in \\
& A_27d. (\forall V3p \in (ty_2Epath_2Epath \ A_27c \ A_27d). ((p \ (ap \ (c_2Epath_2Eis_stopped \\
& A_27c \ A_27d) \ (ap \ (ap \ (ap \ (c_2Epath_2Epcons \ A_27c \ A_27d) \ V1x) \ V2r) \\
& V3p))) \Leftrightarrow False))))))
\end{aligned}$$