

thm_2Epath_2Elabels_LMAP (TMWS36f4SUrN3nefj7ArdYkv3WjdMgGHdv3)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \quad (1)$$

Let $c_2Ellist_2ELMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Ellist_2ELMAP A.27a A.27b \in (((ty_2Ellist_2Ellist A.27b)^{(ty_2Ellist_2Ellist A.27a)})^{(A.27b^{A.27a})}) \quad (2)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (3)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2EFST A.27a A.27b \in (A.27a^{(ty_2Epair_2Eprod A.27a A.27b)}) \quad (4)$$

Let $ty_2Epath_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epath_2Epath A0 A1) \quad (5)$$

Let $c_2Epath_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epath_2EfromPath A.27a A.27b \in ((ty_2Epair_2Eprod A.27a (ty_2Ellist_2Ellist (ty_2Epair_2Eprod A.27b A.27a)))^{(ty_2Epath_2Epath A.27a A.27b)}) \quad (6)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (7)$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a})))$

Definition 4 We define $c_2Epath_2Efirst$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0p \in (ty_2Epath_2Epath\ A_27a\ A_27b)$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b))^{(2^{A_27b})^{A_27a}} \end{aligned} \quad (8)$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2EABS_prod\ x\ y))$

Let $c_2Epath_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2EtoPath \\ A_27a\ A_27b \in ((ty_2Epath_2Epath\ A_27a\ A_27b))^{(ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)))} \end{aligned} \quad (9)$$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A$. **if** $(\exists x \in A. p\ (ap\ P\ x))$ **then** $(the\ (\lambda x. x \in A \wedge p\ (ap\ P\ x)))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a\ P))))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (11)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (12)$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 11 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (14)$$

Definition 12 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (15)$$

Definition 13 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 14 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (16)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (17)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)} \quad (18)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (19)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (20)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (21)$$

Definition 15 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (22)$$

Definition 16 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ x)$

Definition 17 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 18 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap\ (c_2Ebool_2E21\ 2)\ V2t2))))$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in ((ty_2Ellist_2Ellist\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{ty_2Eenum_2Eenum}}) \quad (23)$$

Definition 19 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist_abs\ A_27a)\ h. (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ (V0h\ V1t))$

Definition 20 We define $c_2Epath_2Epcons$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1r \in A_27b. \lambda V2p \in (ty_2Epath_2Epath\ A_27a\ A_27b)\ x. (ap\ (c_2Epath_2Epath\ A_27a\ A_27b)\ (V0x\ V1r\ V2p))$

Definition 21 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone)\ x)$

Definition 22 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2. (ap\ (ap\ (c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E21\ 2)\ V0t))$

Definition 23 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS\ A_27a\ A_27b)\ V0e)$

Definition 24 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (c_2Eoption_2ENONE\ A_27a))$

Definition 25 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ (\lambda V0n \in ty_2Ellist_2Ellist_abs\ A_27a)\ n)$

Definition 26 We define $c_2Epath_2Estopped_at$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. (ap\ (c_2Epath_2Epath\ A_27a\ A_27b)\ V0x)$

Let $c_2Epath_2Elabels : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2Elabels\ A_27a\ A_27b \in ((ty_2Ellist_2Ellist\ A_27b)^{(ty_2Epath_2Epath\ A_27a\ A_27b)}) \quad (24)$$

Definition 27 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Ebool_2E21\ 2)\ V2t))))$

Assume the following.

$$True \quad (25)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in \\ & A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (\\ & ap V0P V1a)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0h \in A.27a. (\forall V1t \in \\ & (ty_2Ellist_2Ellist\ A.27a). ((\neg((ap\ (ap\ (c.2Ellist_2ELCONS\ A.27a) \\ & V0h)\ V1t) = (c.2Ellist_2ELNIL\ A.27a)))) \wedge (\neg((c.2Ellist_2ELNIL \\ & A.27a) = (ap\ (ap\ (c.2Ellist_2ELCONS\ A.27a)\ V0h)\ V1t)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0h1 \in A.27a. (\forall V1t1 \in \\ & (ty_2Ellist_2Ellist\ A.27a). (\forall V2h2 \in A.27a. (\forall V3t2 \in \\ & (ty_2Ellist_2Ellist\ A.27a). ((ap\ (ap\ (c.2Ellist_2ELCONS\ A.27a) \\ & V0h1)\ V1t1) = (ap\ (ap\ (c.2Ellist_2ELCONS\ A.27a)\ V2h2)\ V3t2))) \Leftrightarrow ((\\ & V0h1 = V2h2) \wedge (V1t1 = V3t2)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & (\forall V0f \in (A.27b^{A.27a}). ((ap\ (ap\ (c.2Ellist_2ELMAP\ A.27a\ A.27b) \\ & V0f)\ (c.2Ellist_2ELNIL\ A.27a)) = (c.2Ellist_2ELNIL\ A.27b))) \wedge \\ & (\forall V1f \in (A.27b^{A.27a}). (\forall V2h \in A.27a. (\forall V3t \in \\ & (ty_2Ellist_2Ellist\ A.27a). ((ap\ (ap\ (c.2Ellist_2ELMAP\ A.27a \\ & A.27b)\ V1f)\ (ap\ (ap\ (c.2Ellist_2ELCONS\ A.27a)\ V2h)\ V3t)) = (ap\ (ap \\ & (c.2Ellist_2ELCONS\ A.27b)\ (ap\ V1f\ V2h))\ (ap\ (ap\ (c.2Ellist_2ELMAP \\ & A.27a\ A.27b)\ V1f)\ V3t)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0f \in ((ty_2Ellist_2Ellist\ A.27b)^{A.27a}). (\forall V1g \in \\ & ((ty_2Ellist_2Ellist\ A.27b)^{A.27a}). ((\forall V2x \in A.27a. (((\\ & (ap\ V0f\ V2x) = (c.2Ellist_2ELNIL\ A.27b)) \wedge ((ap\ V1g\ V2x) = (c.2Ellist_2ELNIL \\ & A.27b))) \vee (\exists V3h \in A.27b. (\exists V4y \in A.27a. (((ap\ V0f\ V2x) = \\ & (ap\ (ap\ (c.2Ellist_2ELCONS\ A.27b)\ V3h)\ (ap\ V0f\ V4y))) \wedge ((ap\ V1g\ V2x) = \\ & (ap\ (ap\ (c.2Ellist_2ELCONS\ A.27b)\ V3h)\ (ap\ V1g\ V4y))))))))) \Rightarrow (\forall V5x \in \\ & A.27a. ((ap\ V0f\ V5x) = (ap\ V1g\ V5x)))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in A.27a. (\forall V1y \in A.27b. ((ap\ (c.2Epair_2EFST\ A.27a \\ & A.27b)\ (ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in A.27a. (\forall V1y \in A.27b. ((ap\ (c.2Epair_2ESND\ A.27a \\ & A.27b)\ (ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0a \in (ty_2Epath_2Epath\ A_27a\ A_27b).((ap\ (c_2Epath_2EtoPath \\ & A_27a\ A_27b)\ (ap\ (c_2Epath_2EfromPath\ A_27a\ A_27b)\ V0a)) = V0a)) \wedge \\ & (\forall V1r \in (ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod \\ & A_27b\ A_27a)))) . ((ap\ (c_2Epath_2EfromPath\ A_27a\ A_27b)\ (ap\ (c_2Epath_2EtoPath \\ & A_27a\ A_27b)\ V1r)) = V1r))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0p \in (ty_2Epath_2Epath\ A_27a\ A_27b).((\exists V1x \in A_27a. \\ & (V0p = (ap\ (c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V1x))) \vee (\exists V2x \in \\ & A_27a. (\exists V3r \in A_27b. (\exists V4q \in (ty_2Epath_2Epath\ A_27a \\ & A_27b). (V0p = (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A_27a\ A_27b)\ V2x)\ V3r) \\ & V4q))))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0x \in A_27a. ((ap\ (c_2Epath_2Elabels\ A_27a\ A_27b)\ (ap\ (\\ & c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V0x)) = (c_2Ellist_2ELNIL \\ & A_27b))) \wedge (\forall V1x \in A_27a. (\forall V2r \in A_27b. (\forall V3p \in \\ & (ty_2Epath_2Epath\ A_27a\ A_27b). ((ap\ (c_2Epath_2Elabels\ A_27a \\ & A_27b)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A_27a\ A_27b)\ V1x)\ V2r)\ V3p)) = \\ & (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27b)\ V2r)\ (ap\ (c_2Epath_2Elabels \\ & A_27a\ A_27b)\ V3p)))))) \end{aligned} \quad (46)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (47)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (51)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee (\neg(\\
& p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee \\
& ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{55}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{56}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{57}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow (\\
& \forall V0p \in (ty_2Epath_2Epath \ A.27a \ A.27b). ((ap \ (c_2Epath_2Elabels \\
& A.27a \ A.27b) \ V0p) = (ap \ (ap \ (c_2Ellist_2ELMAP \ (ty_2Epair_2Eprod \\
& A.27b \ A.27a) \ A.27b) \ (c_2Epair_2EFST \ A.27b \ A.27a)) \ (ap \ (c_2Epair_2ESND \\
& A.27a \ (ty_2Ellist_2Ellist \ (ty_2Epair_2Eprod \ A.27b \ A.27a))) \ (\\
& ap \ (c_2Epath_2EfromPath \ A.27a \ A.27b) \ V0p))))
\end{aligned}$$