thm_2Epath_2Elast__pmap (TM-ScC6kavVEJQL71n4PeSAFYGdChVp2jwqN)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Epath\_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epath\_2Epath\ A0\ A1) \quad (1)$$

Let $c\_2Epath\_2Elast : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2Elast\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}) \quad (2)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (3)$$

Let $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \quad (4)$$

Let $c\_2Epath\_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2EfromPath\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Ellist\_2Ellist\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)))^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}) \tag{5}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{6}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{7}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{8}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{9}$$

Definition 7 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 8 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{10}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{11}$$

Definition 9 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{12}$$

Definition 10 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 11 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

Let $ty\_2Eoption\_2Eoption : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{14}$$

Let $c\_2Ellist\_2Ellist\_\_rep : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ellist\_2Ellist\_\_rep\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \tag{15}$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{16}$$

Definition 12 *We define* $\mathsf{c\_2Ebool\_2E\_2F\_5C}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Esum\_2Esum : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}\forall A1.nonempty\ A1{\Rightarrow}nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{17}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{18}$$

Definition 13 *We define* $\mathsf{c\_2Esum\_2EINL}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{19}$$

Definition 14 *We define* $\mathsf{c\_2Eoption\_2ESOME}$ *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 15 *We define* $\mathsf{c\_2Emin\_2E\_40}$ *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \land$
of type $\iota{\Rightarrow}\iota$*.*

Definition 16 *We define* $\mathsf{c\_2Ebool\_2ECOND}$ *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2Ellist\_2Ellist\_\_abs : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ellist\_2Ellist\_\_abs\ A\_27a \in ((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \tag{20}$$

Definition 17 *We define* $\mathsf{c\_2Ellist\_2ELCONS}$ *to be* $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist\ A.$

Definition 18 *We define* $\mathsf{c\_2Ebool\_2E\_3F}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 19 *We define* $\mathsf{c\_2Eone\_2Eone}$ *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 20 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 21 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 22 *We define* c_2Ellist_2ELNIL *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ (\lambda V0n \in ty\_$

Definition 23 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 24 *We define* c_2Ellist_2ELFINITE *to be* $\lambda A\_27a : \iota.(\lambda V0a0 \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ ($

Definition 25 *We define* c_2Epath_2Efinite *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0sigma \in (ty\_2Epath\_2Epath\ A$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{21}$$

Definition 26 *We define* c_2Epath_2Efirst *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (ty\_2Epath\_2Epath\ A\_27a\ A$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{22}$$

Definition 27 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epath\_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2EtoPath\ A\_27a\ A\_27b \in ((ty\_2Epath\_2Epath\ A\_27a\ A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Ellist\_2Ellist\ (ty\_2Epair\_2Eprod} \tag{23}$$

Definition 28 *We define* c_2Epath_2Epcons *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1r \in A\_27b.\lambda V2p$

Definition 29 *We define* c_2Epath_2Estopped__at *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Epath$

Definition 30 *We define* c_2Epair_2E_23_23 *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0f \in (A\_27$

Let $c\_2Ellist\_2ELMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Ellist\_2ELMAP\ A\_27a\ A\_27b \in (((ty\_2Ellist\_2Ellist\ A\_27b)^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{24}$$

Definition 31 *We define* c_2Epath_2Epmap *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0f \in (A\_27$

Assume the following.

$$True \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{28}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{29}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0x \in A\_27a.((ap\ (c\_2Epath\_2Elast\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}\ A\_27a\ A\_27b)\ V0x)) = V0x)) \wedge (\forall V1x \in A\_27a.(\forall V2r \in A\_27b.(\forall V3p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((ap\ (c\_2Epath\_2Elast\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V1x)\ V2r)\ V3p)) = (ap\ (c\_2Epath\_2Elast\ A\_27a\ A\_27b)\ V3p)))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0P \in (2^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}).(((\forall V1x \in A\_27a.(p\ (ap\ V0P\ (ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}\ A\_27a\ A\_27b)\ V1x)))) \wedge (\forall V2x \in A\_27a.(\forall V3r \in A\_27b.(\forall V4p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).(((p\ (ap\ (\mathsf{c\_2Epath\_2Efinite}\ A\_27a\ A\_27b)\ V4p)) \wedge (p\ (ap\ V0P\ V4p))) \Rightarrow (p\ (ap\ V0P\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V2x)\ V3r)\ V4p)))))))) \Rightarrow (\forall V5q \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((p\ (ap\ (\mathsf{c\_2Epath\_2Efinite}\ A\_27a\ A\_27b)\ V5q)) \Rightarrow (p\ (ap\ V0P\ V5q)))))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27c^{A\_27d}).((\forall V2x \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epmap}\ A\_27a\ A\_27d\ A\_27b\ A\_27c)\ V0f)\ V1g)\ (ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}\ A\_27a\ A\_27d)\ V2x)) = (ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}\ A\_27b\ A\_27c)\ (ap\ V0f\ V2x)))) \land (\forall V3x \in A\_27a.(\forall V4r \in A\_27d.(\forall V5p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27d).((ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epmap}\ A\_27a\ A\_27d\ A\_27b\ A\_27c)\ V0f)\ V1g)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27d)\ V3x)\ V4r)\ V5p)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27b\ A\_27c)\ (ap\ V0f\ V3x))\ (ap\ V1g\ V4r))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epmap}\ A\_27a\ A\_27d\ A\_27b\ A\_27c)\ V0f)\ V1g)\ V5p)))))))) \tag{33}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0f \in (A\_27c^{A\_27a}). (\forall V1g \in (A\_27d^{A\_27b}).(\forall V2p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((p\ (ap\ (\mathsf{c\_2Epath\_2Efinite}\ A\_27a\ A\_27b)\ V2p)) \Rightarrow ((ap\ (c\_2Epath\_2Elast\ A\_27c\ A\_27d)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epmap}\ A\_27a\ A\_27b\ A\_27c\ A\_27d)\ V0f)\ V1g)\ V2p)) = (ap\ V0f\ (ap\ (c\_2Epath\_2Elast\ A\_27a\ A\_27b)\ V2p)))))))$$