

thm_2Epath_2Elast__take
(TMEpVyi5i3p7Ag5BnqdeUL6pdfuyyc5vzoX)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $ty_2Epath_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epath_2Epath A0 A1) \tag{1}$$

Let $c_2Epath_2Elast : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epath_2Elast A_27a A_27b \in (A_27a^{(ty_2Epath_2Epath A_27a A_27b)}) \tag{2}$$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty ty_2Eenum_2Eenum \tag{3}$$

Let $c_2Epath_2Eel : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epath_2Eel A_27a A_27b \in ((A_27a^{(ty_2Epath_2Epath A_27a A_27b)})^{ty_2Eenum_2Eenum}) \tag{4}$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \tag{5}$$

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 7 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone$

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E7E$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (6)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (7)$$

Definition 10 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (8)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (9)$$

Definition 11 We define $c_2Eoption_2EONE$ to be $\lambda A_27a : \iota.(ap (c_2Eoption_2Eoption_ABS A_27a) (c$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (11)$$

Definition 12 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 13 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (12)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (13)$$

Definition 14 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Definition 15 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 16 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (15)$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \quad (16)$$

Let $c_2Epath_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2EfromPath\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod\ A_27b\ A_27a)))^{(ty_2Epath_2Epath\ A_27a\ A_27b)}) \quad (17)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (18)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (19)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)}) \quad (21)$$

Definition 17 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS$

Definition 18 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption$

Definition 19 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in ((ty_2Ellist_2Ellist\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum}}) \quad (22)$$

Definition 20 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist\ A_27a)$

Definition 21 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 22 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ (\lambda V0n \in ty_2Ellist_2Ellist\ A_27a))$

Definition 23 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Ebool_2E_3F\ 2)\ V2t))))$

Definition 24 We define $c_2Ellist_2Ellength_rel$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in (ty_2Ellist_2Ellist\ A_27a). (\lambda V1a1 \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ (\lambda V2a2 \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ V2a2))))))$

Definition 25 We define $c_2Ellist_2ELFINITE$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ (\lambda V1a1 \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ (\lambda V2a2 \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ V2a2))))))$

Definition 26 We define $c_2Ellist_2ELLENGTH$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (ap\ (c_2Ellist_2ELFINITE\ A_27a)\ V0ll))$

Let $c_2Eoption_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2ETHE\ A_27a \in (A_27a^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (23)$$

Let $c_2Ellist_2ELTAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2ELTAKE\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Ellist_2Ellist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)^{(ty_2Enum_2Enum)}}) \quad (24)$$

Definition 27 We define $c_2Ellist_2EtoList$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (ap\ (ap\ (c_2Ellist_2ELTAKE\ A_27a)\ V0ll)\ (c_2Ellist_2ELLENGTH\ A_27a))\ V0ll)$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (25)$$

Definition 28 We define $c_2Epath_2Efinite$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0sigma \in (ty_2Epath_2Epath\ A_27a\ A_27b). (ap\ (ap\ (c_2Epath_2ELength\ A_27a\ A_27b)\ V0sigma))$

Definition 29 We define $c_2Epath_2Elength$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0p \in (ty_2Epath_2Epath\ A_27a\ A_27b). (ap\ (c_2Epath_2ELength\ A_27a\ A_27b)\ V0p)$

Definition 30 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap\ (c_2Eprim_rec_2E_3C\ V0m)\ V1n)$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (26)$$

Definition 31 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ V0x\ V1y)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (27)$$

Definition 32 We define c_2Epath_2EPL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0p \in (ty_2Epath_2Epath\ A_27a\ A_27b)$

Let $c_2Epath_2Etail : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2Etail \\ A_27a\ A_27b \in ((ty_2Epath_2Epath\ A_27a\ A_27b)^{(ty_2Epath_2Epath\ A_27a\ A_27b)}) \end{aligned} \quad (28)$$

Let $c_2Epath_2Efirst_label : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2Efirst_label \\ A_27a\ A_27b \in (A_27b^{(ty_2Epath_2Epath\ A_27a\ A_27b)}) \end{aligned} \quad (29)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (30)$$

Definition 33 We define $c_2Epath_2Efirst$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0p \in (ty_2Epath_2Epath\ A_27a\ A_27b)$

Let $c_2Epath_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2EtoPath \\ A_27a\ A_27b \in ((ty_2Epath_2Epath\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)))}) \end{aligned} \quad (31)$$

Definition 34 We define c_2Epath_2Epcns to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1r \in A_27b. \lambda V2p \in A_27b$

Definition 35 We define $c_2Epath_2Estopped_at$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. (ap\ (c_2Epath_2Epath\ A_27a\ A_27b)\ V0x)$

Let $c_2Epath_2Etake : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2Etake \\ A_27a\ A_27b \in (((ty_2Epath_2Epath\ A_27a\ A_27b)^{(ty_2Epath_2Epath\ A_27a\ A_27b)})^{ty_2Enum_2Enum}) \end{aligned} \quad (32)$$

Definition 36 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 37 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 38 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Ebool_2EIN\ A_27a)\ V1s)$

Definition 39 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27a}). (ap\ (c_2Ebool_2EIN\ A_27a)\ V1s)$

Assume the following.

$$True \quad (33)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (40)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (41)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))))) \Rightarrow (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27)))))) \quad (42)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p (ap\ V0P\ V2x)))) \Leftrightarrow (p (ap\ V0P\ V1a)))))) \quad (43)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg((ap\ c_2Enum_2ESUC\ V0n) = c_2Enum_2E0))) \quad (44)$$

Assume the following.

$$(\forall V0P \in (2^{ty_2Enum_2Enum}). (((p (ap\ V0P\ c_2Enum_2E0)) \wedge (\forall V1n \in ty_2Enum_2Enum. ((p (ap\ V0P\ V1n)) \Rightarrow (p (ap\ V0P\ (ap\ c_2Enum_2ESUC\ V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. (p (ap\ V0P\ V2n)))))) \quad (45)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0P \in (2^{(ty_2Epath_2Epath\ A_27a\ A_27b)}). ((\forall V1p \in (ty_2Epath_2Epath\ A_27a\ A_27b). (p (ap\ V0P\ V1p))) \Leftrightarrow ((\forall V2x \in A_27a. (p (ap\ V0P\ (ap (c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V2x)))) \wedge (\forall V3x \in A_27a. (\forall V4r \in A_27b. (\forall V5p \in (ty_2Epath_2Epath\ A_27a\ A_27b). (p (ap\ V0P\ (ap (ap (ap (c_2Epath_2Epcons\ A_27a\ A_27b)\ V3x)\ V4r)\ V5p)))))))))) \quad (46)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in A_27a. ((ap (c_2Epath_2Efirst\ A_27a\ A_27b)\ (ap (c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V0x)) = V0x)) \wedge (\forall V1x \in A_27a. (\forall V2r \in A_27b. (\forall V3p \in (ty_2Epath_2Epath\ A_27a\ A_27b). ((ap (c_2Epath_2Efirst\ A_27a\ A_27b)\ (ap (ap (ap (c_2Epath_2Epcons\ A_27a\ A_27b)\ V1x)\ V2r)\ V3p)) = V1x)))))) \quad (47)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in A_27a. ((ap (c_2Epath_2Elast\ A_27a\ A_27b)\ (ap (c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V0x)) = V0x)) \wedge (\forall V1x \in A_27a. (\forall V2r \in A_27b. (\forall V3p \in (ty_2Epath_2Epath\ A_27a\ A_27b). ((ap (c_2Epath_2Elast\ A_27a\ A_27b)\ (ap (ap (ap (c_2Epath_2Epcons\ A_27a\ A_27b)\ V1x)\ V2r)\ V3p)) = (ap (c_2Epath_2Elast\ A_27a\ A_27b)\ V3p)))))) \quad (48)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in A.27a. (\forall V1r \in A.27b. (\forall V2p \in (ty_2Epath_2Epath \\ & A.27a\ A.27b). ((ap\ (c_2Epath_2Etail\ A.27a\ A.27b)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcns \\ & A.27a\ A.27b)\ V0x)\ V1r)\ V2p)) = V2p)))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in A.27a. (\forall V1r \in A.27b. (\forall V2p \in (ty_2Epath_2Epath \\ & A.27a\ A.27b). ((ap\ (c_2Epath_2Efirst_label\ A.27a\ A.27b)\ (ap\ (\\ & ap\ (ap\ (c_2Epath_2Epcns\ A.27a\ A.27b)\ V0x)\ V1r)\ V2p)) = V1r)))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & (\forall V0p \in (ty_2Epath_2Epath\ A.27a\ A.27b). ((ap\ (ap\ (c_2Epath_2Eel \\ & A.27a\ A.27b)\ c_2Enum_2E0)\ V0p) = (ap\ (c_2Epath_2Efirst\ A.27a\ A.27b)\ \\ & V0p))) \wedge (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in (ty_2Epath_2Epath \\ & A.27a\ A.27b). ((ap\ (ap\ (c_2Epath_2Eel\ A.27a\ A.27b)\ (ap\ c_2Enum_2ESUC \\ & V1n))\ V2p) = (ap\ (ap\ (c_2Epath_2Eel\ A.27a\ A.27b)\ V1n)\ (ap\ (c_2Epath_2Etail \\ & A.27a\ A.27b)\ V2p)))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & (\forall V0x \in A.27a. ((ap\ (c_2Epath_2EPL\ A.27a\ A.27b)\ (ap\ (c_2Epath_2Estopped_at \\ & A.27a\ A.27b)\ V0x)) = (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Enum_2Enum) \\ & c_2Enum_2E0)\ (c_2Epred_set_2EEMPTY\ ty_2Enum_2Enum)))) \wedge (\forall V1x \in \\ & A.27a. (\forall V2r \in A.27b. (\forall V3q \in (ty_2Epath_2Epath\ A.27a \\ & A.27b). ((ap\ (c_2Epath_2EPL\ A.27a\ A.27b)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcns \\ & A.27a\ A.27b)\ V1x)\ V2r)\ V3q)) = (ap\ (ap\ (c_2Epred_set_2EINSERT\ ty_2Enum_2Enum) \\ & c_2Enum_2E0)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ ty_2Enum_2Enum\ ty_2Enum_2Enum) \\ & c_2Enum_2ESUC)\ (ap\ (c_2Epath_2EPL\ A.27a\ A.27b)\ V3q)))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0p \in (ty_2Epath_2Epath\ A.27a\ A.27b). (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & ty_2Enum_2Enum)\ c_2Enum_2E0)\ (ap\ (c_2Epath_2EPL\ A.27a\ A.27b)\ \\ & V0p)))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0p \in (ty_2Epath_2Epath\ A_27a\ A_27b).((ap\ (ap\ (c_2Epath_2Etake \\
& A_27a\ A_27b)\ c_2Enum_2E0)\ V0p) = (ap\ (c_2Epath_2Estopped_at\ A_27a \\
& A_27b)\ (ap\ (c_2Epath_2Efirst\ A_27a\ A_27b)\ V0p)))) \wedge (\forall V1n \in \\
& ty_2Enum_2Enum.(\forall V2p \in (ty_2Epath_2Epath\ A_27a\ A_27b). \\
& ((ap\ (ap\ (c_2Epath_2Etake\ A_27a\ A_27b)\ (ap\ c_2Enum_2ESUC\ V1n)) \\
& V2p) = (ap\ (ap\ (ap\ (c_2Epath_2Epcns\ A_27a\ A_27b)\ (ap\ (c_2Epath_2Efirst \\
& A_27a\ A_27b)\ V2p))\ (ap\ (c_2Epath_2Efirst_label\ A_27a\ A_27b)\ V2p)) \\
& (ap\ (ap\ (c_2Epath_2Etake\ A_27a\ A_27b)\ V1n)\ (ap\ (c_2Epath_2Etail \\
& A_27a\ A_27b)\ V2p))))))))) \\
\end{aligned} \tag{54}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\neg(p\ (ap\ (ap \\
(c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \tag{55}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.(\forall V2s \in (2^{A_27a}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& V0x)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V1y)\ V2s)))) \Leftrightarrow ((V0x = \\
& V1y) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V2s))))))))) \\
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0y \in A_27b.(\forall V1s \in (2^{A_27a}).(\forall V2f \in (A_27b^{A_27a}). \\
& ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\
& A_27a\ A_27b)\ V2f)\ V1s)))) \Leftrightarrow (\exists V3x \in A_27a.((V0y = (ap\ V2f\ V3x)) \wedge \\
& (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s))))))))) \\
\end{aligned} \tag{57}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\
(ap\ c_2Enum_2ESUC\ V0m) = (ap\ c_2Enum_2ESUC\ V1n)) \Leftrightarrow (V0m = V1n)))) \tag{58}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0i \in ty_2Enum_2Enum.(\forall V1p \in (ty_2Epath_2Epath \\
& A_27a\ A_27b).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Enum_2Enum)\ V0i)\ (\\
& ap\ (c_2Epath_2EPL\ A_27a\ A_27b)\ V1p)))) \Rightarrow ((ap\ (c_2Epath_2Elast\ A_27a \\
& A_27b)\ (ap\ (ap\ (c_2Epath_2Etake\ A_27a\ A_27b)\ V0i)\ V1p)) = (ap\ (ap \\
& (c_2Epath_2Eel\ A_27a\ A_27b)\ V0i)\ V1p)))))) \\
\end{aligned}$$