

thm_2Epath_2Elength__pmap (TMRfvKE- jkHQUg7eScGKExQz1oypcMTK6qiF)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x) \text{ of type } \iota \Rightarrow \iota)$.

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x))$.
Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \forall A 1. \text{nonempty } A 1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A 0 \ A 1) \tag{1}$$

Let `ty_2Ellist_2Ellist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \text{nonempty } (\text{ty_2Ellist_2Ellist } A 0) \tag{2}$$

Let `ty_2Epath_2Epath` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \forall A 1. \text{nonempty } A 1 \Rightarrow \text{nonempty } (\text{ty_2Epath_2Epath } A 0 \ A 1) \tag{3}$$

Let `c_2Epath_2EfromPath` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Epath_2EfromPath } A. 27a \ A. 27b \in ((\text{ty_2Epair_2Eprod } A. 27a \ (\text{ty_2Ellist_2Ellist } (\text{ty_2Epair_2Eprod } A. 27b \ A. 27a))) (\text{ty_2Epath_2Epath } A. 27a \ A. 27b)) \tag{4}$$

Let `c_2Epair_2ESND` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Epair_2ESND } A. 27a \ A. 27b \in (A. 27b \ (\text{ty_2Epair_2Eprod } A. 27a \ A. 27b)) \tag{5}$$

Let `c_2Epair_2EFST` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c_2Epair_2EFST } A. 27a \ A. 27b \in (A. 27a \ (\text{ty_2Epair_2Eprod } A. 27a \ A. 27b)) \tag{6}$$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (7)$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 8 We define $c_2Epair_2E_23_23$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f \in (A_27c$

Let $c_2Ellist_2ELMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Ellist_2ELMAP A_27a A_27b \in ((ty_2Ellist_2Ellist A_27b)^{(ty_2Ellist_2Ellist A_27a)}(A_27b^{A_27a})) \quad (8)$$

Let $c_2Epath_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epath_2EtoPath A_27a A_27b \in ((ty_2Epath_2Epath A_27a A_27b)^{(ty_2Epair_2Eprod A_27a (ty_2Ellist_2Ellist (ty_2Epair_2Eprod A_27a A_27b))}) \quad (9)$$

Definition 9 We define c_2Epath_2Epmap to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f \in (A_27c$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (10)$$

Let $c_2Eoption_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2ETHE A_27a \in (A_27a^{(ty_2Eoption_2Eoption A_27a)}) \quad (11)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (12)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Definition 10 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 11 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 12 We define $c_2Epath_2Efirst$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0p \in (ty_2Epath_2Epath A_27a A_27b) . \lambda V0p . V0p$.
Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{14}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{15}$$

Definition 13 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 14 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{16}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{17}$$

Definition 15 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum . (ap\ c_2Enum_2EABS_num\ c_2Enum_2ESUC_REP\ V0m)$.

Definition 16 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum . (ap\ (ap\ c_2Earithmetic_2EZERO\ V0n))\ c_2Enum_2ESUC\ V0n)$.

Definition 17 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum . V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{18}$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a . nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)} \tag{19}$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{20}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0 . nonempty\ A0 \Rightarrow \forall A1 . nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{21}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a . nonempty\ A_27a \Rightarrow \forall A_27b . nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{22}$$

Definition 18 We define c_Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_Esum_2EABS$
Let $c_Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (23)$$

Definition 19 We define $c_Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_Eoption_2Eoption_ABS$
Let $c_Eellist_2Eellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_Eellist_2Eellist_abs A_27a \in ((ty_2Eellist_2Eellist A_27a)^{(ty_2Eoption_2Eoption A_27a)^{ty_2Eenum_2Eenum}}) \quad (24)$$

Definition 20 We define $c_Eellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Eellist_2Eellist A_27a$

Definition 21 We define $c_Epath_2Epathcons$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1r \in A_27b. \lambda V2p$

Definition 22 We define c_Eone_2Eone to be $(ap (c_Emin_2E40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone$

Definition 23 We define c_Ebool_2E7E to be $(\lambda V0t \in 2. (ap (ap c_Emin_2E3D_3D_3E V0t) c_Ebool_2E7E$

Definition 24 We define c_Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_Esum_2EABS$

Definition 25 We define $c_Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_Eoption_2Eoption_ABS A_27a) (c_Eoption_2ENONE$

Definition 26 We define $c_Eellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap (c_Eellist_2Eellist_abs A_27a) (\lambda V0n \in ty_2Eellist_2Eellist$

Definition 27 We define $c_Epath_2Estopped_at$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. (ap (c_Epath_2Estopped_at$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (25)$$

Definition 28 We define c_Ebool_2E3F to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_Emin_2E40$

Definition 29 We define $c_Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_Ebool_2E21 2) (\lambda V2t \in 2. (ap (c_Ebool_2E5C_2F$

Definition 30 We define $c_Eellist_2Ellength_rel$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in (ty_2Eellist_2Eellist A_27a). (\lambda V0a1 \in (ty_2Eellist_2Eellist$

Definition 31 We define $c_Eellist_2ELFINITE$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in (ty_2Eellist_2Eellist A_27a). (ap (c_Eellist_2ELFINITE$

Definition 32 We define $c_Eellist_2ELLENGTH$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Eellist_2Eellist A_27a). (ap (ap (c_Eellist_2ELLENGTH$

Let $c_Eellist_2ELTAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_Eellist_2ELTAKE A_27a \in ((ty_2Eoption_2Eoption (ty_2Elist_2Elist A_27a))^{(ty_2Eellist_2Eellist A_27a)^{ty_2Eenum_2Eenum}}) \quad (26)$$

Definition 33 We define $c_Eellist_2EtoList$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Eellist_2Eellist A_27a). (ap (ap (ap (c_Eellist_2EtoList$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (27)$$

Definition 34 We define $c_2Epath_2Efinite$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0sigma \in (ty_2Epath_2Epath\ A_27a)$

Definition 35 We define $c_2Epath_2Elength$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (ty_2Epath_2Epath\ A_27a)$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & \forall V2p \in ty_2Enum_2Enum.(((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m) \\ & V2p) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V2p)) \Leftrightarrow (V0m = V1n)))))) \end{aligned} \quad (28)$$

Assume the following.

$$True \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True) \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True)))) \end{aligned} \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (36)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p \ V0t))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\
& 2.(((\forall V2x \in A_27a.(p \ (ap \ V0P \ V2x))) \wedge (p \ V1Q)) \Leftrightarrow (\forall V3x \in \\
& A_27a.((p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\
& 2^{A_27a}).((\forall V2x \in A_27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\
& V0P) \vee (\forall V3x \in A_27a.(p \ (ap \ V1Q \ V3x))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\
& 2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))) \Rightarrow \\
& (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\
& (\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a.(((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge ((p \ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \\
& V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ V1Q) \ V3x_27) \\
& V5y_27))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow ((\forall V0t1 \in A_27a.(\forall V1t2 \in \\
& A_27a.((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2ET) \ V0t1) \\
& V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a.(\forall V3t2 \in A_27a.((ap \\
& (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2EF) \ V2t1) \ V3t2) = V3t2))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.(((ap \ (c_2Eoption_2ESOME \ A_27a) \ V0x) = (ap \ (c_2Eoption_2ESOME \\
& A_27a) \ V1y)) \Leftrightarrow (V0x = V1y)))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0P \in (2^{(ty_2Epath_2Epath\ A_27a\ A_27b)}).((\forall V1x \in \\
& A_27a.(p\ (ap\ V0P\ (ap\ (c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V1x)))) \wedge \\
& \quad (\forall V2x \in A_27a.(\forall V3r \in A_27b.(\forall V4p \in (ty_2Epath_2Epath \\
& \quad A_27a\ A_27b).(((p\ (ap\ (c_2Epath_2Efinite\ A_27a\ A_27b)\ V4p)) \wedge (\\
& p\ (ap\ V0P\ V4p))) \Rightarrow (p\ (ap\ V0P\ (ap\ (ap\ (ap\ (c_2Epath_2Epcns\ A_27a\ A_27b) \\
& \quad V2x)\ V3r)\ V4p)))))) \Rightarrow (\forall V5q \in (ty_2Epath_2Epath\ A_27a\ A_27b). \\
& \quad ((p\ (ap\ (c_2Epath_2Efinite\ A_27a\ A_27b)\ V5q)) \Rightarrow (p\ (ap\ V0P\ V5q)))))) \\
& \hspace{15em} (45)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0f \in (A_27b^{A_27a}). \\
& \quad (\forall V1g \in (A_27c^{A_27d}).((\forall V2x \in A_27a.((ap\ (ap\ (ap\ (\\
& c_2Epath_2Epmap\ A_27a\ A_27d\ A_27b\ A_27c)\ V0f)\ V1g)\ (ap\ (c_2Epath_2Estopped_at \\
& \quad A_27a\ A_27d)\ V2x)) = (ap\ (c_2Epath_2Estopped_at\ A_27b\ A_27c)\ (\\
& \quad \quad ap\ V0f\ V2x)))) \wedge (\forall V3x \in A_27a.(\forall V4r \in A_27d.(\forall V5p \in \\
& \quad (ty_2Epath_2Epath\ A_27a\ A_27d).((ap\ (ap\ (ap\ (c_2Epath_2Epmap \\
& \quad A_27a\ A_27d\ A_27b\ A_27c)\ V0f)\ V1g)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcns \\
& \quad A_27a\ A_27d)\ V3x)\ V4r)\ V5p)) = (ap\ (ap\ (ap\ (c_2Epath_2Epcns\ A_27b \\
& \quad A_27c)\ (ap\ V0f\ V3x))\ (ap\ V1g\ V4r))\ (ap\ (ap\ (ap\ (c_2Epath_2Epmap\ A_27a \\
& \quad \quad A_27d\ A_27b\ A_27c)\ V0f)\ V1g)\ V5p))))))))) \\
& \hspace{15em} (46)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0f \in (A_27c^{A_27a}). \\
& \quad (\forall V1g \in (A_27d^{A_27b}).(\forall V2p \in (ty_2Epath_2Epath\ A_27a \\
& A_27b).((p\ (ap\ (c_2Epath_2Efinite\ A_27c\ A_27d)\ (ap\ (ap\ (ap\ (c_2Epath_2Epmap \\
& \quad A_27a\ A_27b\ A_27c\ A_27d)\ V0f)\ V1g)\ V2p))) \Leftrightarrow (p\ (ap\ (c_2Epath_2Efinite \\
& \quad \quad A_27a\ A_27b)\ V2p)))))) \\
& \hspace{15em} (47)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow ((\forall V0x \in A.27a. \\
& \quad ((ap\ (c.2Epath.2Elength\ A.27a\ A.27b)\ (ap\ (c.2Epath.2Estopped_at \\
& \quad A.27a\ A.27b)\ V0x)) = (ap\ (c.2Eoption.2ESOME\ ty.2Enum.2Enum)\ (ap \\
& \quad c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO)))))) \wedge \\
& \quad (\forall V1x \in A.27c. (\forall V2r \in A.27d. (\forall V3p \in (ty.2Epath.2Epath \\
& \quad A.27c\ A.27d). ((ap\ (c.2Epath.2Elength\ A.27c\ A.27d)\ (ap\ (ap\ (ap\ (\\
& \quad c.2Epath.2Epcns\ A.27c\ A.27d)\ V1x)\ V2r)\ V3p)) = (ap\ (ap\ (ap\ (c.2Ebool.2ECOND \\
& \quad (ty.2Eoption.2Eoption\ ty.2Enum.2Enum))\ (ap\ (c.2Epath.2Efinite \\
& \quad A.27c\ A.27d)\ V3p))\ (ap\ (c.2Eoption.2ESOME\ ty.2Enum.2Enum)\ (ap \\
& \quad (ap\ c.2Earithmetic.2E.2B\ (ap\ (c.2Eoption.2ETHE\ ty.2Enum.2Enum) \\
& \quad (ap\ (c.2Epath.2Elength\ A.27c\ A.27d)\ V3p)))\ (ap\ c.2Earithmetic.2ENUMERAL \\
& \quad (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO))))))\ (c.2Eoption.2ENONE \\
& \quad ty.2Enum.2Enum))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0p \in (ty.2Epath.2Epath\ A.27a\ A.27b). (((p\ (ap\ (c.2Epath.2Efinite \\
& \quad A.27a\ A.27b)\ V0p)) \Leftrightarrow (\exists V1n \in ty.2Enum.2Enum. ((ap\ (c.2Epath.2Elength \\
& \quad A.27a\ A.27b)\ V0p) = (ap\ (c.2Eoption.2ESOME\ ty.2Enum.2Enum)\ V1n)))) \wedge \\
& \quad ((\neg(p\ (ap\ (c.2Epath.2Efinite\ A.27a\ A.27b)\ V0p))) \Leftrightarrow ((ap\ (c.2Epath.2Elength \\
& \quad A.27a\ A.27b)\ V0p) = (c.2Eoption.2ENONE\ ty.2Enum.2Enum))))))
\end{aligned} \tag{49}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{50}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{51}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \tag{52}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \tag{53}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{59}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{62}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{63}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{64}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0f \in (A_27b^{A_27a}). \\ & (\forall V1g \in (A_27d^{A_27c}). (\forall V2p \in (ty_2Epath_2Epath\ A_27a \\ & A_27c). ((ap\ (c_2Epath_2Elength\ A_27b\ A_27d)\ (ap\ (ap\ (ap\ (c_2Epath_2Epmap \\ & A_27a\ A_27c\ A_27b\ A_27d)\ V0f)\ V1g)\ V2p))) = (ap\ (c_2Epath_2Elength \\ & A_27a\ A_27c)\ V2p)))))) \end{aligned}$$