

# thm\_2Epath\_2Enumeral\_drop (TMSth- BCK4jtSapm9LK2asQAKzpJXFDVePw4)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$\text{nonempty } ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E21$  to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ (ap\ (c\_2Emin\_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num\ (c\_2Enum\_2E0\ m))$

Let  $c\_2Earithmetic\_2E2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 6** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EBIT2 V0n))$

**Definition 7** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (7)$$

**Definition 8** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EBIT1 V0n))$

**Definition 9** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 10** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

Let  $ty\_2Epath\_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epath\_2Epath A0 A1) \quad (8)$$

Let  $c\_2Epath\_2Etail : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epath\_2Etail A\_27a A\_27b \in ((ty\_2Epath\_2Epath A\_27a A\_27b)^{(ty\_2Epath\_2Epath A\_27a A\_27b)}) \quad (9)$$

Let  $c\_2Epath\_2Edrop : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epath\_2Edrop A\_27a A\_27b \in (((ty\_2Epath\_2Epath A\_27a A\_27b)^{(ty\_2Epath\_2Epath A\_27a A\_27b)})^{ty\_2Enum\_2Enum}) \quad (10)$$

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))))$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0f \in ((A\_27a)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}). \\ & \quad (\forall V1g \in (A\_27a)^{ty\_2Enum\_2Enum}). ((\forall V2n \in ty\_2Enum\_2Enum. \\ & \quad ((ap V1g (ap c\_2Enum\_2ESUC V2n)) = (ap (ap V0f V2n) (ap c\_2Enum\_2ESUC V2n)))) \Leftrightarrow ((\forall V3n \in ty\_2Enum\_2Enum. ((ap V1g (ap c\_2Earithmetic\_2ENUMERAL \\ & \quad (ap c\_2Earithmetic\_2EBIT1 V3n))) = (ap (ap V0f (ap (ap c\_2Earithmetic\_2E\_2D \\ & \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V3n))) \\ & \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \\ & \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 V3n)))))) \wedge \\ & \quad (\forall V4n \in ty\_2Enum\_2Enum. ((ap V1g (ap c\_2Earithmetic\_2ENUMERAL \\ & \quad (ap c\_2Earithmetic\_2EBIT2 V4n))) = (ap (ap V0f (ap c\_2Earithmetic\_2ENUMERAL \\ & \quad (ap c\_2Earithmetic\_2EBIT1 V4n))) (ap c\_2Earithmetic\_2ENUMERAL \\ & \quad (ap c\_2Earithmetic\_2EBIT2 V4n))))))))) \quad (11) \end{aligned}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p V0t1) \Rightarrow (p V1t2)) \Rightarrow ((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))) \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\ & (\forall V0p \in (ty\_2Epath\_2Epath A.27a A.27b). ((ap (ap (c\_2Epath\_2Edrop \\ & A.27a A.27b) c\_2Enum\_2E0) V0p) = V0p)) \wedge (\forall V1n \in ty\_2Enum\_2Enum. \\ & (\forall V2p \in (ty\_2Epath\_2Epath A.27a A.27b). ((ap (ap (c\_2Epath\_2Edrop \\ & A.27a A.27b) (ap c\_2Enum\_2ESUC V1n)) V2p) = (ap (ap (c\_2Epath\_2Edrop \\ & A.27a A.27b) V1n) (ap (c\_2Epath\_2Etail A.27a A.27b) V2p)))))) \quad (13) \end{aligned}$$

**Theorem 1**

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\ & (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1p \in (ty\_2Epath\_2Epath \\ & A.27a A.27b). ((ap (ap (c\_2Epath\_2Edrop A.27a A.27b) (ap c\_2Earithmetic\_2ENUMERAL \\ & (ap c\_2Earithmetic\_2EBIT1 V0n))) V1p) = (ap (ap (c\_2Epath\_2Edrop \\ & A.27a A.27b) (ap (ap c\_2Earithmetic\_2E\_2D (ap c\_2Earithmetic\_2ENUMERAL \\ & (ap c\_2Earithmetic\_2EBIT1 V0n))) (ap c\_2Earithmetic\_2ENUMERAL \\ & (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) (ap (c\_2Epath\_2Etail \\ & A.27a A.27b) V1p)))))) \wedge (\forall V2n \in ty\_2Enum\_2Enum. (\forall V3p \in \\ & (ty\_2Epath\_2Epath A.27a A.27b). ((ap (ap (c\_2Epath\_2Edrop A.27a \\ & A.27b) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 \\ & V2n))) V3p) = (ap (ap (c\_2Epath\_2Edrop A.27a A.27b) (ap c\_2Earithmetic\_2ENUMERAL \\ & (ap c\_2Earithmetic\_2EBIT1 V2n))) (ap (c\_2Epath\_2Etail A.27a A.27b) \\ & V3p)))))) \end{aligned}$$