

thm\_2Epath\_2Eokpath\_drop  
(TMNjGGPTtVXdeodSeezZ5YoazKCiqKn38Fh)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2E$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow P \Rightarrow Q)$  of type  $\iota$ .

Let  $ty\_2Eenum\_2Eenum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eenum\_2Eenum \tag{1}$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{2}$$

**Definition 6** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p))$  of type  $\iota \Rightarrow \iota$ .

**Definition 7** We define  $c\_2Eone\_2Eone$  to be  $(ap (c\_2Emin\_2E\_40 ty\_2Eone\_2Eone) (\lambda V0x \in ty\_2Eone\_2Eone$

**Definition 8** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{3}$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \tag{4}$$

**Definition 10** We define  $c\_Esum\_2EINR$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27b. (ap (c\_Esum\_2EABS$   
Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Eoption\_2Eoption A0) \quad (5)$$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS A\_27a \in ((ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Esum\_2Esum A\_27a ty\_2Eone\_2Eone)}) \quad (6)$$

**Definition 11** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota. (ap (c\_2Eoption\_2Eoption\_ABS A\_27a) (c$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (7)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (8)$$

**Definition 12** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 13** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (10)$$

**Definition 14** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 15** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap (ap c\_2Earithmetic$

**Definition 16** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (12)$$

Let  $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Ellist\_2Ellist A0) \quad (13)$$

Let  $ty\_2Epath\_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epath\_2Epath\ A0\ A1) \quad (14)$$

Let  $c\_2Epath\_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2EfromPath\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Ellist\_2Ellist\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)))^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}) \quad (15)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \quad (16)$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (17)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (18)$$

Let  $c\_2Ellist\_2Ellist\_rep : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_rep\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \quad (19)$$

**Definition 17** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27a. (ap\ (c\_2Esum\_2EABS$

**Definition 18** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. (ap\ (c\_2Eoption\_2Eoption\_$

**Definition 19** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

Let  $c\_2Ellist\_2Ellist\_abs : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_abs\ A\_27a \in ((ty\_2Ellist\_2Ellist\ A\_27a)^{(ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum}}) \quad (20)$$

**Definition 20** We define  $c\_2Ellist\_2ELCONS$  to be  $\lambda A\_27a : \iota. \lambda V0h \in A\_27a. \lambda V1t \in (ty\_2Ellist\_2Ellist\ A$

**Definition 21** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 22** We define  $c\_2Ellist\_2ELNIL$  to be  $\lambda A\_27a : \iota. (ap\ (c\_2Ellist\_2Ellist\_abs\ A\_27a)\ (\lambda V0n \in ty$

**Definition 23** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 24** We define  $c\_2Ellist\_2Ellength\_rel$  to be  $\lambda A\_27a : \iota. (\lambda V0a0 \in (ty\_2Ellist\_2Ellist\ A\_27a). (\lambda V$

**Definition 25** We define  $c\_2Ellist\_2ELFINITE$  to be  $\lambda A\_27a : \iota. (\lambda V0a0 \in (ty\_2Ellist\_2Ellist\ A\_27a). (ap\ (c$

**Definition 26** We define  $c\_2Ellist\_2ELLENGTH$  to be  $\lambda A\_27a : \iota. \lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a). (ap\ (a$

Let  $c\_2Eoption\_2ETHE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2ETHE\ A\_27a \in (A\_27a^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (21)$$

Let  $c\_2Ellist\_2ELTAKE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2ELTAKE\ A\_27a \in (((ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist\ A\_27a))^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (22)$$

**Definition 27** We define  $c\_2Ellist\_2EtoList$  to be  $\lambda A\_27a : \iota. \lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a). (ap\ (ap\ (ap$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (23)$$

**Definition 28** We define  $c\_2Epath\_2Efinite$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0sigma \in (ty\_2Epath\_2Epath\ A$

**Definition 29** We define  $c\_2Epath\_2Elength$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0p \in (ty\_2Epath\_2Epath\ A\_27a$

**Definition 30** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (24)$$

**Definition 31** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap\ (c\_2$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \quad (25)$$

**Definition 32** We define  $c\_2Epath\_2EPL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0p \in (ty\_2Epath\_2Epath\ A\_27a\ A$

Let  $c\_2Epath\_2Etail : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2Etail\ A\_27a\ A\_27b \in ((ty\_2Epath\_2Epath\ A\_27a\ A\_27b)^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}) \quad (26)$$

Let  $c\_2Epath\_2Edrop : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$A\_27a\ A\_27b \in (((ty\_2Epath\_2Epath\ A\_27a\ A\_27b)^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)})^{ty\_2Enum\_2Enum}) \quad (27)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (28)$$

**Definition 33** We define  $c\_2Epath\_2Efirst$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b)$

Let  $c\_2Epath\_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2EtoPath \\ A\_27a\ A\_27b \in ((ty\_2Epath\_2Epath\ A\_27a\ A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Ellist\_2Ellist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))})) \end{aligned} \quad (29)$$

**Definition 34** We define  $c\_2Epath\_2Eprecons$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1r \in A\_27b.\lambda V2p \in A\_27c$

**Definition 35** We define  $c\_2Epath\_2Estopped\_at$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Epath\_2EtoPath\ A\_27a\ A\_27b)\ V0x)$

**Definition 36** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

**Definition 37** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c)^{A\_27a \times A\_27b})$

**Definition 38** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EIN)$ .

**Definition 39** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2Epred\_set\_2EIN\ A\_27a)\ V1t)$

**Definition 40** We define  $c\_2Epath\_2Eokpath\_f$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in (((2^{A\_27a})^{A\_27b})^{A\_27a})$

**Definition 41** We define  $c\_2Epred\_set\_2ESUBSET$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2Epred\_set\_2EIN\ A\_27a)\ V1t)$

**Definition 42** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_set\_2EIN\ A\_27a)\ V0P)$

**Definition 43** We define  $c\_2EfixedPoint\_2Egfp$  to be  $\lambda A\_27a : \iota.\lambda V0f \in ((2^{A\_27a})^{(2^{A\_27a})}).(ap\ (c\_2Epred\_set\_2EIN\ A\_27a)\ V0f)$

**Definition 44** We define  $c\_2Epath\_2Eokpath$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in (((2^{A\_27a})^{A\_27b})^{A\_27a})$

**Definition 45** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EIN)$ .

**Definition 46** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_2Epred\_set\_2EIN\ A\_27a)\ V1s)$

**Definition 47** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A\_27b}).(ap\ (c\_2Epred\_set\_2EIN\ A\_27b)\ V1s)$

Assume the following.

$$True \quad (30)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (31)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p V0t)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in \\ & 2. (((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1a \in \\ & A.27a. ((\exists V2x \in A.27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p ( \\ & ap V0P V1a)))))) \end{aligned} \quad (40)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg((ap\ c\_2Enum\_2ESUC\ V0n) = c\_2Enum\_2E0))) \quad (41)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}). (((p\ (ap\ V0P\ c\_2Enum\_2E0)) \wedge (\forall V1n \in ty\_2Enum\_2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c\_2Enum\_2ESUC\ V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum. (p\ (ap\ V0P\ V2n)))))) \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0P \in (2^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}). ((\forall V1p \in \\ (ty\_2Epath\_2Epath\ A\_27a\ A\_27b). (p\ (ap\ V0P\ V1p))) \Leftrightarrow ((\forall V2x \in \\ A\_27a. (p\ (ap\ V0P\ (ap\ (c\_2Epath\_2Estopped\_at\ A\_27a\ A\_27b)\ V2x)))) \wedge \\ (\forall V3x \in A\_27a. (\forall V4r \in A\_27b. (\forall V5p \in (ty\_2Epath\_2Epath \\ A\_27a\ A\_27b). (p\ (ap\ V0P\ (ap\ (ap\ (ap\ (c\_2Epath\_2Epcns\ A\_27a\ A\_27b) \\ V3x)\ V4r)\ V5p)))))))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0x \in A\_27a. (\forall V1r \in A\_27b. (\forall V2p \in (ty\_2Epath\_2Epath \\ A\_27a\ A\_27b). ((ap\ (c\_2Epath\_2Etail\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (c\_2Epath\_2Epcns \\ A\_27a\ A\_27b)\ V0x)\ V1r)\ V2p)) = V2p)))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ (\forall V0x \in A\_27a. ((ap\ (c\_2Epath\_2EPL\ A\_27a\ A\_27b)\ (ap\ (c\_2Epath\_2Estopped\_at \\ A\_27a\ A\_27b)\ V0x)) = (ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ ty\_2Enum\_2Enum) \\ c\_2Enum\_2E0)\ (c\_2Epred\_set\_2EEMPTY\ ty\_2Enum\_2Enum)))) \wedge (\forall V1x \in \\ A\_27a. (\forall V2r \in A\_27b. (\forall V3q \in (ty\_2Epath\_2Epath\ A\_27a \\ A\_27b). ((ap\ (c\_2Epath\_2EPL\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (c\_2Epath\_2Epcns \\ A\_27a\ A\_27b)\ V1x)\ V2r)\ V3q)) = (ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ ty\_2Enum\_2Enum) \\ c\_2Enum\_2E0)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum) \\ c\_2Enum\_2ESUC)\ (ap\ (c\_2Epath\_2EPL\ A\_27a\ A\_27b)\ V3q)))))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b). (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ ty\_2Enum\_2Enum)\ c\_2Enum\_2E0)\ (ap\ (c\_2Epath\_2EPL\ A\_27a\ A\_27b) \\ V0p)))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& (\forall V0p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((ap\ (ap\ (c\_2Epath\_2Edrop \\
& A\_27a\ A\_27b)\ c\_2Enum\_2E0)\ V0p) = V0p)) \wedge (\forall V1n \in ty\_2Enum\_2Enum. \\
& (\forall V2p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((ap\ (ap\ (c\_2Epath\_2Edrop \\
& A\_27a\ A\_27b)\ (ap\ c\_2Enum\_2ESUC\ V1n))\ V2p) = (ap\ (ap\ (c\_2Epath\_2Edrop \\
& A\_27a\ A\_27b)\ V1n)\ (ap\ (c\_2Epath\_2Etail\ A\_27a\ A\_27b)\ V2p)))))) \\
& \hspace{10em} (47)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \forall V0R \in (((2^{A\_27a})^{A\_27b})^{A\_27a}).((\forall V1x \in A\_27a.( \\
& p\ (ap\ (ap\ (c\_2Epath\_2Eokpath\ A\_27a\ A\_27b)\ V0R)\ (ap\ (c\_2Epath\_2Estopped\_at \\
& A\_27a\ A\_27b)\ V1x)))) \wedge (\forall V2x \in A\_27a.(\forall V3r \in A\_27b. \\
& (\forall V4p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((p\ (ap\ (ap\ (c\_2Epath\_2Eokpath \\
& A\_27a\ A\_27b)\ V0R)\ (ap\ (ap\ (ap\ (c\_2Epath\_2Epcons\ A\_27a\ A\_27b)\ V2x) \\
& V3r)\ V4p))) \Leftrightarrow ((p\ (ap\ (ap\ (ap\ V0R\ V2x)\ V3r)\ (ap\ (c\_2Epath\_2Efirst\ A\_27a \\
& A\_27b)\ V4p))) \wedge (p\ (ap\ (ap\ (c\_2Epath\_2Eokpath\ A\_27a\ A\_27b)\ V0R)\ V4p))))))))) \\
& \hspace{10em} (48)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg(p\ (ap\ (ap \\
& (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (c\_2Epred\_set\_2EEMPTY\ A\_27a)))))) \\
& \hspace{10em} (49)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\
& A\_27a.(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\
& V0x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ A\_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = \\
& V1y) \vee (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ V2s)))))) \\
& \hspace{10em} (50)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \forall V0y \in A\_27b.(\forall V1s \in (2^{A\_27a}).(\forall V2f \in (A\_27b^{A\_27a}). \\
& ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27b)\ V0y)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\
& A\_27a\ A\_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A\_27a.((V0y = (ap\ V2f\ V3x)) \wedge \\
& (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V1s)))))) \\
& \hspace{10em} (51)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\
& ((ap\ c\_2Enum\_2ESUC\ V0m) = (ap\ c\_2Enum\_2ESUC\ V1n)) \Leftrightarrow (V0m = V1n)))) \\
& \hspace{10em} (52)
\end{aligned}$$



**Theorem 1**

$$\begin{aligned} & \forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow ( \\ & \quad \forall V0R \in (((2^{A_{27a}})^{A_{27b}})^{A_{27a}}).(\forall V1p \in (ty\_2Epath\_2Epath \\ & \quad A_{27a}\ A_{27b}).(\forall V2i \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ & \quad ty\_2Enum\_2Enum)\ V2i)\ (ap\ (c\_2Epath\_2EPL\ A_{27a}\ A_{27b})\ V1p)))) \wedge ( \\ & \quad p\ (ap\ (ap\ (c\_2Epath\_2Eokpath\ A_{27a}\ A_{27b})\ V0R)\ V1p))) \Rightarrow (p\ (ap\ (ap \\ & \quad (c\_2Epath\_2Eokpath\ A_{27a}\ A_{27b})\ V0R)\ (ap\ (ap\ (c\_2Epath\_2Edrop \\ & \quad A_{27a}\ A_{27b})\ V2i)\ V1p)))))) \end{aligned}$$