

thm_2Epath_2Eokpath__parallel__comp
(TMZ31ttoFq3pA9pxSvdD1vGq4J22jH34iBG)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x) \text{ of type } \iota \Rightarrow \iota).$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define `c_2Ecombin_2EK` to be $\lambda A. \lambda 27a : \iota. \lambda A. \lambda 27b : \iota. (\lambda V0x \in A. \lambda V1y \in A. \lambda V0x \in A. \lambda V1y \in A. V0x)$

Definition 5 We define `c_2Ecombin_2ES` to be $\lambda A. \lambda 27a : \iota. \lambda A. \lambda 27b : \iota. \lambda A. \lambda 27c : \iota. (\lambda V0f \in ((A. \lambda 27c^{A.27b})^{A.27a}))$

Definition 6 We define `c_2Ecombin_2EI` to be $\lambda A. \lambda 27a : \iota. (\text{ap } (\text{ap } (\text{c_2Ecombin_2ES } A. \lambda 27a (A. \lambda 27a^{A.27a})) A. \lambda 27a))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A0 \ A1) \tag{1}$$

Let `c_2Epair_2EFST` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. \lambda 27a. \text{nonempty } A. \lambda 27a \Rightarrow \forall A. \lambda 27b. \text{nonempty } A. \lambda 27b \Rightarrow \text{c_2Epair_2EFST } A. \lambda 27a \ A. \lambda 27b \in (A. \lambda 27a^{(\text{ty_2Epair_2Eprod } A. \lambda 27a \ A. \lambda 27b)}) \tag{2}$$

Let `c_2Epair_2ESND` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. \lambda 27a. \text{nonempty } A. \lambda 27a \Rightarrow \forall A. \lambda 27b. \text{nonempty } A. \lambda 27b \Rightarrow \text{c_2Epair_2ESND } A. \lambda 27a \ A. \lambda 27b \in (A. \lambda 27b^{(\text{ty_2Epair_2Eprod } A. \lambda 27a \ A. \lambda 27b)}) \tag{3}$$

Definition 7 We define `c_2Ebool_2E_3F` to be $\lambda A. \lambda 27a : \iota. (\lambda V0P \in (2^{A.27a}). (\text{ap } V0P \ (\text{ap } (\text{c_2Emin_2E_40 } A. \lambda 27a))))$

Let `ty_2Ellist_2Ellist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Ellist_2Ellist } A0) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{omega}) \quad (12)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (13)$$

Definition 14 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (14)$$

Definition 15 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Epath_2Etail : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2Etail \\ A_27a\ A_27b \in ((ty_2Epath_2Epath\ A_27a\ A_27b)^{(ty_2Epath_2Epath\ A_27a\ A_27b)}) \end{aligned} \quad (15)$$

Definition 16 We define $c_2Epath_2Efirst$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (ty_2Epath_2Epath\ A_27a\ A_27b)$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (16)$$

Definition 17 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone))$

Definition 18 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 19 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (17)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (18)$$

Definition 20 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum\ V0e\ A_27a))$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (19)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (20)$$

Definition 34 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 35 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. (c_2Ebool_2E_5C_2F t1 t2))))))$.

Definition 36 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Epred_set_2EUNION s t))$.

Definition 37 We define $c_2Epath_2Eokpath_f$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in (((2^{A_27a})^{A_27b})^{A_27a})$.

Definition 38 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Epred_set_2ESUBSET s t))$.

Definition 39 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap (c_2Epred_set_2EBIGUNION P))$.

Definition 40 We define $c_2EfixedPoint_2Egfp$ to be $\lambda A_27a : \iota. \lambda V0f \in ((2^{A_27a})^{(2^{A_27a})}). (ap (c_2EfixedPoint_2Egfp f))$.

Definition 41 We define $c_2Epath_2Eokpath$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in (((2^{A_27a})^{A_27b})^{A_27a})$.

Let $c_2Epath_2Edrop : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epath_2Edrop \\ & A_27a A_27b \in (((ty_2Epath_2Epath A_27a A_27b)^{(ty_2Epath_2Epath A_27a A_27b)})^{ty_2Enum_2Enum}) \end{aligned} \quad (26)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (27)$$

Definition 42 We define $c_2Ellist_2Ellength_rel$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in (ty_2Ellist_2Ellist A_27a). (\lambda V1a1 \in (ty_2Ellist_2Ellist A_27a). (c_2Ellist_2Ellength_rel a0 a1)))$.

Definition 43 We define $c_2Ellist_2ELFINITE$ to be $\lambda A_27a : \iota. (\lambda V0a0 \in (ty_2Ellist_2Ellist A_27a). (ap (c_2Ellist_2ELFINITE a0)))$.

Definition 44 We define $c_2Ellist_2ELLENGTH$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist A_27a). (ap (c_2Ellist_2ELLENGTH ll))$.

Let $c_2Eoption_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2ETHE A_27a \in (A_27a)^{(ty_2Eoption_2Eoption A_27a)} \quad (28)$$

Let $c_2Ellist_2ELTAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Ellist_2ELTAKE A_27a \in (((ty_2Eoption_2Eoption \\ & (ty_2Elist_2Elist A_27a))^{(ty_2Ellist_2Ellist A_27a)})^{ty_2Enum_2Enum}) \end{aligned} \quad (29)$$

Definition 45 We define $c_2Ellist_2EtoList$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist A_27a). (ap (ap (c_2Ellist_2EtoList ll)))$.

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ELENGTH A_27a \in (ty_2Enum_2Enum)^{(ty_2Elist_2Elist A_27a)} \quad (30)$$

Definition 46 We define $c_2Epath_2Efinite$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0sigma \in (ty_2Epath_2Epath A_27a A_27b)$.

Definition 47 We define $c_2Epath_2Elength$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (ty_2Epath_2Epath A_27a$

Definition 48 We define $c_2Eprim_rec_2E3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 49 We define c_2Epath_2EPL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (ty_2Epath_2Epath A_27a A_27b$

Let $c_2Epath_2Eparallel_comp : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\ & nonempty A_27c \Rightarrow \forall A_27d.nonempty A_27d \Rightarrow \forall A_27e.nonempty \\ & A_27e \Rightarrow c_2Epath_2Eparallel_comp A_27a A_27b A_27c A_27d A_27e \in \\ & (((((2^{(ty_2Epair_2Eprod A_27c A_27e)} A_27b)^{(ty_2Epair_2Eprod A_27a A_27d)}))^{((2^{A_27e})^{A_27b})^{A_27d}}))^{((2^{A_27c})^{A_27b})^{A_27e}}) \end{aligned} \quad (31)$$

Assume the following.

$$True \quad (32)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True)))) \end{aligned} \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (41)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (42)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a. (p\ (ap\ V1Q\ V4x))))))) \quad (43)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. (p\ (ap\ V1Q\ V2x))) \Leftrightarrow (\forall V3x \in A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \quad (44)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q)) \Leftrightarrow ((\forall V3x \in A_27a. (p\ (ap\ V1P\ V3x)) \vee (p\ V0Q)))))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge ((p\ V2C) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (47)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (48)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \Rightarrow (49)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.((ap (c_{.2}Ecombin_{.2}EI A_{.27a}) V0x) = V0x)) \quad (50)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in (ty_{.2}Epair_{.2}Eprod A_{.27a} A_{.27b}).(\exists V1q \in A_{.27a}.(\exists V2r \in A_{.27b}.(V0x = (ap (ap (c_{.2}Epair_{.2}E_{.2}C A_{.27a} A_{.27b}) V1q) V2r)))))) \quad (51)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.((ap (c_{.2}Epair_{.2}EFST A_{.27a} A_{.27b}) (ap (ap (c_{.2}Epair_{.2}E_{.2}C A_{.27a} A_{.27b}) V0x) V1y)) = V0x))) \quad (52)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.((ap (c_{.2}Epair_{.2}ESND A_{.27a} A_{.27b}) (ap (ap (c_{.2}Epair_{.2}E_{.2}C A_{.27a} A_{.27b}) V0x) V1y)) = V1y))) \quad (53)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1r \in A_{.27b}.(\forall V2p \in (ty_{.2}Epath_{.2}Epath A_{.27a} A_{.27b}).(\forall V3y \in A_{.27a}.(\forall V4s \in A_{.27b}.(\forall V5q \in (ty_{.2}Epath_{.2}Epath A_{.27a} A_{.27b}).(((ap (ap (ap (c_{.2}Epath_{.2}Epcns A_{.27a} A_{.27b}) V0x) V1r) V2p) = (ap (ap (ap (c_{.2}Epath_{.2}Epcns A_{.27a} A_{.27b}) V3y) V4s) V5q)) \Leftrightarrow ((V0x = V3y) \wedge ((V1r = V4s) \wedge (V2p = V5q)))))))))) \quad (54)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27a}.(\forall V2r \in A_{.27b}.(\forall V3p \in (ty_{.2}Epath_{.2}Epath A_{.27a} A_{.27b}).((\neg((ap (c_{.2}Epath_{.2}Estopped_{.2}at A_{.27a} A_{.27b}) V0x) = (ap (ap (ap (c_{.2}Epath_{.2}Epcns A_{.27a} A_{.27b}) V1y) V2r) V3p)))) \wedge (\neg((ap (ap (ap (c_{.2}Epath_{.2}Epcns A_{.27a} A_{.27b}) V1y) V2r) V3p) = (ap (c_{.2}Epath_{.2}Estopped_{.2}at A_{.27a} A_{.27b}) V0x)))))))))) \quad (55)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0p \in (ty.2Epath.2Epath\ A.27a\ A.27b).((\exists V1x \in A.27a. \\
& \quad (V0p = (ap\ (c.2Epath.2Estopped_at\ A.27a\ A.27b)\ V1x))) \vee (\exists V2x \in \\
& \quad A.27a.(\exists V3r \in A.27b.(\exists V4q \in (ty.2Epath.2Epath\ A.27a \\
& \quad A.27b).(V0p = (ap\ (ap\ (ap\ (c.2Epath.2Epcns\ A.27a\ A.27b)\ V2x)\ V3r \\
& \quad V4q))))))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow (\forall V0f \in (A.27b^{A.27a}). \\
& \quad (\forall V1g \in (A.27c^{A.27d}).((\forall V2x \in A.27a.((ap\ (ap\ (ap\ (\\
& \quad c.2Epath.2Emap\ A.27a\ A.27d\ A.27b\ A.27c)\ V0f)\ V1g)\ (ap\ (c.2Epath.2Estopped_at \\
& \quad A.27a\ A.27d)\ V2x)) = (ap\ (c.2Epath.2Estopped_at\ A.27b\ A.27c)\ (\\
& \quad ap\ V0f\ V2x)))))) \wedge (\forall V3x \in A.27a.(\forall V4r \in A.27d.(\forall V5p \in \\
& \quad (ty.2Epath.2Epath\ A.27a\ A.27d).((ap\ (ap\ (ap\ (c.2Epath.2Emap \\
& \quad A.27a\ A.27d\ A.27b\ A.27c)\ V0f)\ V1g)\ (ap\ (ap\ (ap\ (c.2Epath.2Epcns \\
& \quad A.27a\ A.27d)\ V3x)\ V4r)\ V5p)) = (ap\ (ap\ (ap\ (c.2Epath.2Epcns\ A.27b \\
& \quad A.27c)\ (ap\ V0f\ V3x))\ (ap\ V1g\ V4r))\ (ap\ (ap\ (ap\ (c.2Epath.2Emap\ A.27a \\
& \quad A.27d\ A.27b\ A.27c)\ V0f)\ V1g)\ V5p))))))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow (\forall V0f \in (A.27c^{A.27a}). \\
& \quad (\forall V1g \in (A.27d^{A.27b}).(\forall V2p \in (ty.2Epath.2Epath\ A.27a \\
& \quad A.27b).((ap\ (c.2Epath.2Efirst\ A.27c\ A.27d)\ (ap\ (ap\ (ap\ (c.2Epath.2Emap \\
& \quad A.27a\ A.27b\ A.27c\ A.27d)\ V0f)\ V1g)\ V2p)) = (ap\ V0f\ (ap\ (c.2Epath.2Efirst \\
& \quad A.27a\ A.27b)\ V2p))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0x \in A.27a.(\forall V1r \in A.27b.(\forall V2p \in (ty.2Epath.2Epath \\
& \quad A.27a\ A.27b).((ap\ (c.2Epath.2Etail\ A.27a\ A.27b)\ (ap\ (ap\ (ap\ (c.2Epath.2Epcns \\
& \quad A.27a\ A.27b)\ V0x)\ V1r)\ V2p)) = V2p))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0p \in (ty.2Epath.2Epath\ A.27a\ A.27b).(p\ (ap\ (ap\ (c.2Ebool.2EIN \\
& \quad ty.2Enum.2Enum)\ c.2Enum.2E0)\ (ap\ (c.2Epath.2EPL\ A.27a\ A.27b) \\
& \quad V0p))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& (\forall V0p \in (ty_2Epath_2Epath\ A.27a\ A.27b).((ap\ (ap\ (c_2Epath_2Edrop \\
& A.27a\ A.27b)\ c_2Enum_2E0)\ V0p) = V0p)) \wedge (\forall V1n \in ty_2Enum_2Enum. \\
& (\forall V2p \in (ty_2Epath_2Epath\ A.27a\ A.27b).((ap\ (ap\ (c_2Epath_2Edrop \\
& A.27a\ A.27b)\ (ap\ c_2Enum_2ESUC\ V1n))\ V2p) = (ap\ (ap\ (c_2Epath_2Edrop \\
& A.27a\ A.27b)\ V1n)\ (ap\ (c_2Epath_2Etail\ A.27a\ A.27b)\ V2p)))))) \\
& \hspace{15em} (61)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0i \in ty_2Enum_2Enum. (\forall V1p \in (ty_2Epath_2Epath \\
& A.27a\ A.27b).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Enum_2Enum)\ (ap\ (ap \\
& c_2Earithmetic_2E_2B\ V0i)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))))))\ (ap\ (c_2Epath_2EPL\ A.27a\ A.27b)\ V1p))) \Rightarrow \\
& ((ap\ (c_2Epath_2Etail\ A.27a\ A.27b)\ (ap\ (ap\ (c_2Epath_2Edrop\ A.27a \\
& A.27b)\ V0i)\ V1p)) = (ap\ (ap\ (c_2Epath_2Edrop\ A.27a\ A.27b)\ (ap\ (ap \\
& c_2Earithmetic_2E_2B\ V0i)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))))))\ V1p)))))) \\
& \hspace{15em} (62)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0R \in (((2^{A.27a})^{A.27b})^{A.27a}). (\forall V1P \in (2^{(ty_2Epath_2Epath\ A.27a\ A.27b)}). \\
& ((\forall V2x \in A.27a. (\forall V3r \in A.27b. (\forall V4p \in (ty_2Epath_2Epath \\
& A.27a\ A.27b). ((p\ (ap\ V1P\ (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A.27a\ A.27b) \\
& V2x)\ V3r)\ V4p))) \Rightarrow ((p\ (ap\ (ap\ (ap\ V0R\ V2x)\ V3r)\ (ap\ (c_2Epath_2Efirst \\
& A.27a\ A.27b)\ V4p))) \wedge (p\ (ap\ V1P\ V4p)))))) \Rightarrow (\forall V5p \in (ty_2Epath_2Epath \\
& A.27a\ A.27b). ((p\ (ap\ V1P\ V5p)) \Rightarrow (p\ (ap\ (ap\ (c_2Epath_2Eokpath\ A.27a \\
& A.27b)\ V0R)\ V5p)))))) \\
& \hspace{15em} (63)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0R \in (((2^{A.27a})^{A.27b})^{A.27a}). ((\forall V1x \in A.27a. (\\
& p\ (ap\ (ap\ (c_2Epath_2Eokpath\ A.27a\ A.27b)\ V0R)\ (ap\ (c_2Epath_2Estopped_at \\
& A.27a\ A.27b)\ V1x)))) \wedge (\forall V2x \in A.27a. (\forall V3r \in A.27b. \\
& (\forall V4p \in (ty_2Epath_2Epath\ A.27a\ A.27b). ((p\ (ap\ (ap\ (c_2Epath_2Eokpath \\
& A.27a\ A.27b)\ V0R)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A.27a\ A.27b)\ V2x) \\
& V3r)\ V4p))) \Leftrightarrow ((p\ (ap\ (ap\ (ap\ V0R\ V2x)\ V3r)\ (ap\ (c_2Epath_2Efirst\ A.27a \\
& A.27b)\ V4p))) \wedge (p\ (ap\ (ap\ (c_2Epath_2Eokpath\ A.27a\ A.27b)\ V0R)\ V4p)))))) \\
& \hspace{15em} (64)
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0R \in (((2^{A.27a})^{A.27b})^{A.27a}).(\forall V1p \in (ty_2Epath_2Epath \\ & A.27a\ A.27b).(\forall V2i \in ty_2Enum_2Enum.(((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & ty_2Enum_2Enum)\ V2i)\ (ap\ (c_2Epath_2EPL\ A.27a\ A.27b)\ V1p)))) \wedge (\\ & p\ (ap\ (ap\ (c_2Epath_2Eokpath\ A.27a\ A.27b)\ V0R)\ V1p))) \Rightarrow (p\ (ap\ (ap \\ & (c_2Epath_2Eokpath\ A.27a\ A.27b)\ V0R)\ (ap\ (ap\ (c_2Epath_2Edrop \\ & A.27a\ A.27b)\ V2i)\ V1p)))))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0n \in ty_2Enum_2Enum.(\forall V1p \in (ty_2Epath_2Epath \\ & A.27a\ A.27b).(\forall V2h \in A.27a.(\forall V3l \in A.27b.(\forall V4t \in \\ & (ty_2Epath_2Epath\ A.27a\ A.27b).(((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Enum_2Enum) \\ & V0n)\ (ap\ (c_2Epath_2EPL\ A.27a\ A.27b)\ V1p)))) \wedge ((ap\ (ap\ (c_2Epath_2Edrop \\ & A.27a\ A.27b)\ V0n)\ V1p) = (ap\ (ap\ (ap\ (c_2Epath_2Econs\ A.27a\ A.27b) \\ & V2h)\ V3l)\ V4t)))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Enum_2Enum)\ (ap \\ & (ap\ c_2Earithmetic_2E_2B\ V0n)\ (ap\ c_2Earithmetic_2ENUMERAL\ (\\ & ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))\ (ap\ (c_2Epath_2EPL \\ & A.27a\ A.27b)\ V1p)))))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow \forall A.27e.nonempty \\ & A.27e \Rightarrow (\forall V0m1 \in (((2^{A.27c})^{A.27b})^{A.27a}).(\forall V1m2 \in \\ & (((2^{A.27e})^{A.27b})^{A.27d}).(\forall V2s1 \in A.27a.(\forall V3s2 \in \\ & A.27d.(\forall V4l \in A.27b.(\forall V5s1.27 \in A.27c.(\forall V6s2.27 \in \\ & A.27e.((p\ (ap\ (ap\ (ap\ (ap\ (ap\ (c_2Epath_2Eparallel_comp\ A.27a \\ & A.27b\ A.27c\ A.27d\ A.27e)\ V0m1)\ V1m2)\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a \\ & A.27d)\ V2s1)\ V3s2))\ V4l)\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27c\ A.27e)\ V5s1.27) \\ & V6s2.27))) \Leftrightarrow ((p\ (ap\ (ap\ (ap\ V0m1\ V2s1)\ V4l)\ V5s1.27)) \wedge (p\ (ap\ (ap\ (\\ & ap\ V1m2\ V3s2)\ V4l)\ V6s2.27)))))) \end{aligned} \quad (67)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (68)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (69)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (70)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (71)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (72)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (74)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (75)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (76)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (77)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (78)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (79)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (80)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q)))) \quad (81)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (82)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow \forall A.27c. \\ & nonempty \ A.27c \Rightarrow (\forall V0p \in (ty_2Epath_2Epath \ (ty_2Epair_2Eprod \\ & \quad A.27a \ A.27b) \ A.27c).(\forall V1m1 \in (((2^{A.27a})^{A.27c})^{A.27a}). \\ & \quad (\forall V2m2 \in (((2^{A.27b})^{A.27c})^{A.27b}).((p \ (ap \ (ap \ (c_2Epath_2Eokpath \\ & \quad (ty_2Epair_2Eprod \ A.27a \ A.27b) \ A.27c) \ (ap \ (ap \ (c_2Epath_2Eparallel_comp \\ & \quad A.27a \ A.27c \ A.27a \ A.27b \ A.27b) \ V1m1) \ V2m2)) \ V0p)) \Leftrightarrow ((p \ (ap \ (ap \ (c_2Epath_2Eokpath \\ & \quad A.27a \ A.27c) \ V1m1) \ (ap \ (ap \ (ap \ (c_2Epath_2Epmap \ (ty_2Epair_2Eprod \\ & \quad A.27a \ A.27b) \ A.27c \ A.27a \ A.27c) \ (c_2Epair_2EFST \ A.27a \ A.27b)) \ (\\ & \quad \lambda V3x \in A.27c.V3x)) \ V0p))) \wedge (p \ (ap \ (ap \ (c_2Epath_2Eokpath \ A.27b \\ & \quad A.27c) \ V2m2) \ (ap \ (ap \ (ap \ (c_2Epath_2Epmap \ (ty_2Epair_2Eprod \ A.27a \\ & \quad A.27b) \ A.27c \ A.27b \ A.27c) \ (c_2Epair_2ESND \ A.27a \ A.27b)) \ (\lambda V4x \in \\ & \quad A.27c.V4x)) \ V0p))))))))) \end{aligned}$$