

thm_2Epath_2Eokpath__plink (TMajF7dQbPAFrSMYtsDD3qL5chL9wi5qgoU)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x))$

Definition 3 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P x) \text{ then } (\text{the } (\lambda x. x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define `c_2Ebool_2E_3F` to be $\lambda A. \lambda 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } V 0P (\text{ap } (\text{c_2Emin_2E_40 } A) P)))$

Definition 5 We define `c_2Ecombin_2E_2EK` to be $\lambda A. \lambda 27a : \iota. \lambda A. \lambda 27b : \iota. (\lambda V 0x \in A. \lambda V 1y \in A. \lambda V 0x \in A. \lambda V 1y \in A. \text{V } 0x \text{ V } 1y)$

Definition 6 We define `c_2Ecombin_2E_2ES` to be $\lambda A. \lambda 27a : \iota. \lambda A. \lambda 27b : \iota. \lambda A. \lambda 27c : \iota. (\lambda V 0f \in ((A. \lambda 27c^{A-27b})^{A-27a}))$

Definition 7 We define `c_2Ecombin_2E_2EI` to be $\lambda A. \lambda 27a : \iota. (\text{ap } (\text{ap } (\text{c_2Ecombin_2E_2ES } A. \lambda 27a (A. \lambda 27a^{A-27a}) A. \lambda 27a)))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \forall A 1. \text{nonempty } A 1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A 0 A 1) \quad (1)$$

Let `ty_2Ellist_2Ellist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \text{nonempty } (\text{ty_2Ellist_2Ellist } A 0) \quad (2)$$

Let `ty_2Epath_2Epath` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \forall A 1. \text{nonempty } A 1 \Rightarrow \text{nonempty } (\text{ty_2Epath_2Epath } A 0 A 1) \quad (3)$$

Let `c_2Epath_2EfromPath` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. \lambda 27a. \text{nonempty } A. \lambda 27a \Rightarrow \forall A. \lambda 27b. \text{nonempty } A. \lambda 27b \Rightarrow \text{c_2Epath_2EfromPath } A. \lambda 27a A. \lambda 27b \in ((\text{ty_2Epair_2Eprod } A. \lambda 27a (\text{ty_2Ellist_2Ellist } (\text{ty_2Epair_2Eprod } A. \lambda 27b A. \lambda 27a)))^{(\text{ty_2Epath_2Epath } A. \lambda 27a A. \lambda 27b)}) \quad (4)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (5)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (6)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 9 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (10)$$

Definition 10 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a}))\ V0P)\ A_27a))$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ V0m)$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 12 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic_2E2B\ V0n))$

Definition 13 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Earithmetic_2E2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (12)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (13)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)}) \quad (14)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (15)$$

Definition 14 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 15 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in 2.))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (16)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (17)$$

Definition 16 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS_sum A_27a A_27b) V0e)$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (18)$$

Definition 17 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap (c_2Eoption_2Eoption_ABS A_27a) V0x)$

Definition 18 We define c_Ebool_2EF to be $(ap (c_Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 19 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.))$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ellist_2Ellist_abs A_27a \in ((ty_2Ellist_2Ellist A_27a)^{(ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum}}) \quad (19)$$

Definition 20 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota.\lambda V0h \in A_27a.\lambda V1t \in (ty_2Ellist_2Ellist A_27a) V0h$

Definition 21 We define c_2Eone_2Eone to be $(ap (c_Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone V0x))$

Definition 22 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E V0t) c_Ebool_2E_7E V0t))$

Definition 23 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS_sum A_27a A_27b) V0e)$

Definition 24 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap (c_2Eoption_2Eoption_ABS A_27a) V0e)$

Definition 25 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota.(ap (c_2Ellist_2Ellist_abs A_27a) (\lambda V0n \in ty_2Ellist_2Ellist A_27a V0n))$

Definition 26 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t \in 2.))$

Definition 27 We define $c_2Ellist_2ELFINITE$ to be $\lambda A_27a : \iota.(\lambda V0a0 \in (ty_2Ellist_2Ellist A_27a).(ap (c_2Ellist_2Ellist A_27a) V0a0))$

Definition 28 We define $c_2Epath_2Efinite$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0sigma \in (ty_2Epath_2Epath A_27a A_27b) V0sigma$

Definition 29 We define c_Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Let $c_Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (20)$$

Definition 30 We define $c_Epath_2Efirst$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0p \in (ty_2Epath_2Epath\ A_27a\ A_27b)$

Let $c_Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (21)$$

Definition 31 We define $c_Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epath_2EtoPath\ x\ y))$

Let $c_Epath_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epath_2EtoPath \\ A_27a\ A_27b \in ((ty_2Epath_2Epath\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)))}) \end{aligned} \quad (22)$$

Definition 32 We define $c_Epath_2Eacons$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1r \in A_27b. \lambda V2p \in (c_2Epath_2EtoPath\ x\ r)$

Definition 33 We define $c_Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a})^{A_27b})$

Let $c_Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (23)$$

Definition 34 We define $c_Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2ET)$.

Definition 35 We define $c_Epath_2Estopped_at$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. (ap\ (c_2Epath_2EtoPath\ x\ x))$

Definition 36 We define $c_Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Epred_set_2EUNION\ s\ t))$

Definition 37 We define $c_Epath_2Eokpath_f$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in (((2^{A_27a})^{A_27b})^{A_27a})$.

Definition 38 We define $c_Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Epred_set_2ESUBSET\ s\ t))$

Definition 39 We define $c_Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap\ (c_2Epred_set_2EBIGUNION\ P))$

Definition 40 We define $c_EfixedPoint_2Egfp$ to be $\lambda A_27a : \iota. \lambda V0f \in ((2^{A_27a})^{(2^{A_27a})}). (ap\ (c_2Epred_set_2EBIGUNION\ f))$

Definition 41 We define $c_Epath_2Eokpath$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in (((2^{A_27a})^{A_27b})^{A_27a})$.

Let $c_2Epath_2Eplink : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2Eplink \\ A_27a\ A_27b \in & (((ty_2Epath_2Epath\ A_27a\ A_27b)^{(ty_2Epath_2Epath\ A_27a\ A_27b)})^{(ty_2Epath_2Epath\ A_27a\ A_27b)}) \end{aligned} \quad (24)$$

Let $c_2Epath_2Elast : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2Elast \\ A_27a\ A_27b \in & (A_27a^{(ty_2Epath_2Epath\ A_27a\ A_27b)}) \end{aligned} \quad (25)$$

Assume the following.

$$True \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ & A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \\ & True)) \end{aligned} \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\forall V1x \in A_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\exists V2x \in A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \wedge (\exists V3x \in A_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge (((\neg(p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B)))))) \quad (42)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (43)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (44)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI\ A_27a)\ V0x) = V0x)) \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0x \in A_27a. ((ap\ (c_2Epath_2Elast\ A_27a\ A_27b)\ (ap\ (c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V0x)) = V0x)) \wedge (\forall V1x \in A_27a. (\forall V2r \in A_27b. \\ & (\forall V3p \in (ty_2Epath_2Epath\ A_27a\ A_27b). ((ap\ (c_2Epath_2Elast\ A_27a\ A_27b)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A_27a\ A_27b)\ V1x)\ V2r)\ V3p)) = (ap\ (c_2Epath_2Elast\ A_27a\ A_27b)\ V3p)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0P \in (2^{(ty_2Epath_2Epath\ A_27a\ A_27b)}). ((\forall V1x \in \\ & A_27a. (p\ (ap\ V0P\ (ap\ (c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V1x)))) \wedge \\ & (\forall V2x \in A_27a. (\forall V3r \in A_27b. (\forall V4p \in (ty_2Epath_2Epath\ A_27a\ A_27b). ((p\ (ap\ (c_2Epath_2Efinite\ A_27a\ A_27b)\ V4p)) \wedge \\ & p\ (ap\ V0P\ V4p))) \Rightarrow (p\ (ap\ V0P\ (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A_27a\ A_27b)\ V2x)\ V3r)\ V4p)))))) \Rightarrow (\forall V5q \in (ty_2Epath_2Epath\ A_27a\ A_27b). \\ & ((p\ (ap\ (c_2Epath_2Efinite\ A_27a\ A_27b)\ V5q)) \Rightarrow (p\ (ap\ V0P\ V5q)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0R \in (((2^{A_27a})^{A_27b})^{A_27a}). ((\forall V1x \in A_27a. (\\ & p\ (ap\ (ap\ (c_2Epath_2Eokpath\ A_27a\ A_27b)\ V0R)\ (ap\ (c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V1x)))) \wedge (\forall V2x \in A_27a. (\forall V3r \in A_27b. \\ & (\forall V4p \in (ty_2Epath_2Epath\ A_27a\ A_27b). ((p\ (ap\ (ap\ (c_2Epath_2Eokpath\ A_27a\ A_27b)\ V0R)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A_27a\ A_27b)\ V2x)\ V3r)\ V4p))) \Leftrightarrow ((p\ (ap\ (ap\ (ap\ V0R\ V2x)\ V3r)\ (ap\ (c_2Epath_2Efirst\ A_27a\ A_27b)\ V4p))) \wedge (p\ (ap\ (ap\ (c_2Epath_2Eokpath\ A_27a\ A_27b)\ V0R)\ V4p)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0x \in A_27a. (\forall V1p \in (ty_2Epath_2Epath\ A_27a\ A_27b). \\ & ((ap\ (ap\ (c_2Epath_2Eplink\ A_27a\ A_27b)\ (ap\ (c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V0x))\ V1p) = V1p))) \wedge (\forall V2x \in A_27a. (\forall V3r \in \\ & A_27b. (\forall V4p1 \in (ty_2Epath_2Epath\ A_27a\ A_27b). (\forall V5p2 \in \\ & (ty_2Epath_2Epath\ A_27a\ A_27b). ((ap\ (ap\ (c_2Epath_2Eplink\ A_27a\ A_27b)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A_27a\ A_27b)\ V2x)\ V3r)\ V4p1))\ V5p2) = (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A_27a\ A_27b)\ V2x)\ V3r)\ (ap\ (\\ & ap\ (c_2Epath_2Eplink\ A_27a\ A_27b)\ V4p1)\ V5p2)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0p1 \in (ty_2Epath_2Epath\ A_27a\ A_27b).(\forall V1p2 \in (\\
& \quad ty_2Epath_2Epath\ A_27a\ A_27b).(((ap\ (c_2Epath_2Elast\ A_27a\ A_27b) \\
V0p1) = (ap\ (c_2Epath_2Efirst\ A_27a\ A_27b)\ V1p2)) \Rightarrow ((ap\ (c_2Epath_2Efirst \\
A_27a\ A_27b)\ (ap\ (ap\ (c_2Epath_2Eplink\ A_27a\ A_27b)\ V0p1)\ V1p2)) = \\
(ap\ (c_2Epath_2Efirst\ A_27a\ A_27b)\ V0p1)))))) \\
\end{aligned} \tag{50}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{51}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\
\end{aligned} \tag{54}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \\
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\
& \quad (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \\
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\
& \quad ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \\
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{60}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0R \in (((2^{A_27a})^{A_27b})^{A_27a}). (\forall V1p1 \in (ty_2Epath_2Epath \\
& A_27a\ A_27b). (\forall V2p2 \in (ty_2Epath_2Epath\ A_27a\ A_27b). (\\
& ((p (ap (c_2Epath_2Efinite\ A_27a\ A_27b)\ V1p1)) \wedge ((ap (c_2Epath_2Elast \\
& A_27a\ A_27b)\ V1p1) = (ap (c_2Epath_2Efirst\ A_27a\ A_27b)\ V2p2))) \Rightarrow \\
& ((p (ap (ap (c_2Epath_2Eokpath\ A_27a\ A_27b)\ V0R) (ap (ap (c_2Epath_2Eplink \\
& A_27a\ A_27b)\ V1p1)\ V2p2))) \Leftrightarrow ((p (ap (ap (c_2Epath_2Eokpath\ A_27a \\
& A_27b)\ V0R)\ V1p1)) \wedge (p (ap (ap (c_2Epath_2Eokpath\ A_27a\ A_27b)\ V0R) \\
& V2p2))))))
\end{aligned}$$