

thm_2Epath_2Eokpath__thm (TMPoxS8fL1xWtF2ezr3toN2p9q1xyg78HAx)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \tag{2}$$

Let $ty_2Epath_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epath_2Epath A0 A1) \tag{3}$$

Let $c_2Epath_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty A.\lambda 27a \Rightarrow \forall A.\lambda 27b.nonempty A.\lambda 27b \Rightarrow c_2Epath_2EfromPath A.27a A.27b \in ((ty_2Epair_2Eprod A.27a (ty_2Ellist_2Ellist (ty_2Epair_2Eprod A.27b A.27a)))^{(ty_2Epath_2Epath A.27a A.27b)}) \tag{4}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty A.\lambda 27a \Rightarrow \forall A.\lambda 27b.nonempty A.\lambda 27b \Rightarrow c_2Epair_2EFST A.27a A.27b \in (A.27a^{(ty_2Epair_2Eprod A.27a A.27b)}) \tag{5}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (15)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in \\ ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)} \quad (16)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (17)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (18)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (19)$$

Definition 15 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in \\ ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone}) \quad (20)$$

Definition 16 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Definition 17 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \mathbf{if}\ (\exists x \in A. p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x. x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 18 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap\ (c_2Emin_2E_40\ A)\ (V1t1\ V2t2))))$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in \\ ((ty_2Ellist_2Ellist\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum}}) \quad (21)$$

Definition 19 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist\ A_27a)$

Let $c_2Epath_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2EtoPath \\ A_27a\ A_27b \in & ((ty_2Epath_2Epath\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod\ (22) \end{aligned}$$

Definition 20 We define c_2Epath_2Epcns to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1r \in A_27b. \lambda V2p$

Definition 21 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone))$

Definition 22 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS$

Definition 23 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (c$

Definition 24 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ (\lambda V0n \in ty$

Definition 25 We define $c_2Epath_2Estopped_at$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. (ap\ (c_2Epath$

Definition 26 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 27 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 28 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x))$

Definition 29 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in & ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})} \end{aligned} \tag{23}$$

Definition 30 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E_2ET)$.

Definition 31 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c$

Definition 32 We define $c_2Epath_2Eokpath_f$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in (((2^{A_27a})^{A_27b})^{A_27a})$.

Definition 33 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ ($

Definition 34 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap\ (c_2Epred_set$

Definition 35 We define $c_2EfixedPoint_2Egfp$ to be $\lambda A_27a : \iota. \lambda V0f \in ((2^{A_27a})^{(2^{A_27a})}). (ap\ (c_2Epred_set$

Definition 36 We define $c_2Epath_2Eokpath$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in (((2^{A_27a})^{A_27b})^{A_27a})$.

Assume the following.

$$True \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0a \in A_27a.(\exists V1x \in A_27a.(V1x = V0a))) \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in \\ A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p (ap\ V0P\ V2x)))) \Leftrightarrow (p (\\ ap\ V0P\ V1a)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0x \in A_27a. (\forall V1y \in A_27a. (((ap\ (c_2Epath_2Estopped_at \\ A_27a\ A_27b)\ V0x) = (ap\ (c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V1y)) \Leftrightarrow \\ (V0x = V1y)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0x \in A_27a. (\forall V1r \in A_27b. (\forall V2p \in (ty_2Epath_2Epath \\ A_27a\ A_27b). (\forall V3y \in A_27a. (\forall V4s \in A_27b. (\forall V5q \in \\ (ty_2Epath_2Epath\ A_27a\ A_27b). (((ap\ (ap\ (ap\ (c_2Epath_2Epcns \\ A_27a\ A_27b)\ V0x)\ V1r)\ V2p) = (ap\ (ap\ (ap\ (c_2Epath_2Epcns\ A_27a \\ A_27b)\ V3y)\ V4s)\ V5q)) \Leftrightarrow ((V0x = V3y) \wedge ((V1r = V4s) \wedge (V2p = V5q)))))))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0x \in A_27a. (\forall V1y \in A_27a. (\forall V2r \in A_27b. (\forall V3p \in \\ (ty_2Epath_2Epath\ A_27a\ A_27b). ((\neg((ap\ (c_2Epath_2Estopped_at \\ A_27a\ A_27b)\ V0x) = (ap\ (ap\ (ap\ (c_2Epath_2Epcns\ A_27a\ A_27b)\ V1y) \\ V2r)\ V3p))) \wedge (\neg((ap\ (ap\ (ap\ (c_2Epath_2Epcns\ A_27a\ A_27b)\ V1y) \\ V2r)\ V3p) = (ap\ (c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V0x)))))))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0R \in (((2^{A_27a})^{A_27b})^{A_27a}). (\forall V1x \in (ty_2Epath_2Epath \\ A_27a\ A_27b). ((p\ (ap\ (ap\ (c_2Epath_2Eokpath\ A_27a\ A_27b)\ V0R)\ V1x)) \Leftrightarrow \\ ((\exists V2x_27 \in A_27a. (V1x = (ap\ (c_2Epath_2Estopped_at\ A_27a \\ A_27b)\ V2x_27))) \vee (\exists V3x_27 \in A_27a. (\exists V4r \in A_27b. \\ (\exists V5p \in (ty_2Epath_2Epath\ A_27a\ A_27b). ((V1x = (ap\ (ap\ (ap \\ (c_2Epath_2Epcns\ A_27a\ A_27b)\ V3x_27)\ V4r)\ V5p)) \wedge ((p\ (ap\ (ap\ (\\ ap\ V0R\ V3x_27)\ V4r) (ap\ (c_2Epath_2Efirst\ A_27a\ A_27b)\ V5p)))) \wedge (\\ p\ (ap\ (ap\ (c_2Epath_2Eokpath\ A_27a\ A_27b)\ V0R)\ V5p)))))))))) \end{aligned} \quad (38)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0R \in (((2^{A_27a})^{A_27b})^{A_27a}).((\forall V1x \in A_27a.(\\ p\ (ap\ (ap\ (c_2Epath_2Eokpath\ A_27a\ A_27b)\ V0R)\ (ap\ (c_2Epath_2Estopped_at \\ & \quad A_27a\ A_27b)\ V1x)))) \wedge (\forall V2x \in A_27a.(\forall V3r \in A_27b. \\ & \quad (\forall V4p \in (ty_2Epath_2Epath\ A_27a\ A_27b).((p\ (ap\ (ap\ (c_2Epath_2Eokpath \\ & \quad A_27a\ A_27b)\ V0R)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A_27a\ A_27b)\ V2x) \\ & \quad V3r)\ V4p)))) \Leftrightarrow ((p\ (ap\ (ap\ (ap\ V0R\ V2x)\ V3r)\ (ap\ (c_2Epath_2Efirst\ A_27a \\ & \quad A_27b)\ V4p)))) \wedge (p\ (ap\ (ap\ (c_2Epath_2Eokpath\ A_27a\ A_27b)\ V0R)\ V4p))))))))) \end{aligned}$$