

thm_2Epath_2Epath__Axiom (TMQw3o5qznzho3c8REnjYre9mesDSZsZ1Ho7)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ecombin_2E_2K$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 4 We define $c_2Ecombin_2E_2S$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 5 We define $c_2Ecombin_2E_2I$ to be $\lambda A.\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2E_2S A_27a (A_27a^{A_27a})) A_27a))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a})).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 7 We define $c_2Ecombin_2E_2Eo$ to be $\lambda A.\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in (A_27b^{A_27c})).\lambda V1g$

Let $c_2Enum_2E_2ZERO_2E_2REP : \iota$ be given. Assume the following.

$$c_2Enum_2E_2ZERO_2E_2REP \in \omega \tag{1}$$

Let $ty_2Enum_2E_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2E_2Enum \tag{2}$$

Let $c_2Enum_2E_2ABS_2E_2num : \iota$ be given. Assume the following.

$$c_2Enum_2E_2ABS_2E_2num \in (ty_2Enum_2E_2Enum^{\omega}) \tag{3}$$

Definition 8 We define $c_2Enum_2E_2E0$ to be $(ap\ c_2Enum_2E_2ABS_2E_2num\ c_2Enum_2E_2ZERO_2E_2REP)$.

Definition 9 We define $c_2Earithmetic_2E_2ZERO$ to be $c_2Enum_2E_2E0$.

Let $c_2Enum_2E_2ERP_2E_2num : \iota$ be given. Assume the following.

$$c_2Enum_2E_2ERP_2E_2num \in (\omega^{ty_2Enum_2E_2Enum}) \tag{4}$$

Let $c_2Enum_2E_2SUC_2E_2REP : \iota$ be given. Assume the following.

$$c_2Enum_2E_2SUC_2E_2REP \in (\omega^{\omega}) \tag{5}$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (6)$$

Definition 11 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 12 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (7)$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \quad (8)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)} \quad (9)$$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in ((ty_2Ellist_2Ellist\ A_27a)^{((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})}) \quad (10)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (11)$$

Definition 13 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 14 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (12)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (13)$$

Definition 15 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (14)$$

Definition 16 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ x)$

Definition 17 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A)\ P)$ of type $\iota \Rightarrow \iota$.

Definition 18 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone))$

Definition 19 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 20 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2EF))$

Definition 21 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS\ A_27a\ A_27b)\ V0e)$

Definition 22 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (c_2Emin_2E_40\ A_27a))$

Definition 23 We define $c_2Ellist_2ELHD$ to be $\lambda A_27a : \iota.\lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a).(ap\ (ap\ (c_2Emin_2E_40\ A_27a)\ V0ll))$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2Eoption_CASE\ A_27a\ A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (15)$$

Definition 24 We define $c_2Ellist_2ELTL$ to be $\lambda A_27a : \iota.\lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a).(ap\ (ap\ (ap\ (c_2Emin_2E_40\ A_27a)\ V0ll)\ A_27a))$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_MAP\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27b)^{(ty_2Eoption_2Eoption\ A_27a)})^{(A_27b^{A_27a})}) \quad (16)$$

Definition 25 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a)\ V0P)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (17)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (18)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (19)$$

Definition 26 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a})$

Definition 27 We define $c_2Epair_2Epair_CASE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0p \in (ty_2Epair_2Eprod$

Definition 28 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap (c_2Ellist_2Ellist_abs A_27a) (\lambda V0n \in ty_2Ellist_2Ellist$

Let $ty_2Epath_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epath_2Epath A0 A1) \quad (20)$$

Let $c_2Epath_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epath_2EfromPath A_27a A_27b \in ((ty_2Epair_2Eprod A_27a (ty_2Ellist_2Ellist (ty_2Epair_2Eprod A_27b A_27a)))^{(ty_2Epath_2Epath A_27a A_27b)}) \quad (21)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (22)$$

Definition 29 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 30 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist A_27a$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (23)$$

Definition 31 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2Eprod$

Let $c_2Epath_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epath_2EtoPath A_27a A_27b \in ((ty_2Epath_2Epath A_27a A_27b)^{(ty_2Epair_2Eprod A_27a (ty_2Ellist_2Ellist (ty_2Epair_2Eprod A_27b A_27a))}) \quad (24)$$

Definition 32 We define $c_2Epath_2Efirst$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0p \in (ty_2Epath_2Epath A_27a A_27b$

Definition 33 We define c_2Epath_2Epcns to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1r \in A_27b. \lambda V2p \in (ty_2Epath_2Epath$

Definition 34 We define $c_2Epath_2Estopped_at$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. (ap (c_2Epath_2Epath$

Definition 35 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. (\lambda V3t3 \in 2. ($

Assume the following.

$$True \quad (25)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee \neg(p V0t))) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2. (\neg(p V0t) \Rightarrow ((p V0t) \Rightarrow False))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2. (\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (35)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (36)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t))))) \quad (38)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\neg(\exists V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x))))) \quad (39)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27a}).((\forall V2x \in A.27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A.27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A.27a.(p (ap V1Q V4x))))))) \quad (40)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.(((\exists V2x \in A.27a.(p (ap V0P V2x))) \vee (p V1Q)) \Leftrightarrow (\exists V3x \in A.27a.((p (ap V0P V3x)) \vee (p V1Q))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee ((p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C))))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))) \wedge (((\neg(p V0A) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))) \quad (44)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p (ap\ V0P\ V2x)))) \Leftrightarrow (p (ap\ V0P\ V1a)))))) \quad (45)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0P \in ((2^{A_27b})^{A_27a}). ((\forall V1x \in A_27a. (\exists V2y \in A_27b. (p (ap (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A_27b^{A_27a}). (\forall V4x \in A_27a. (p (ap (ap\ V0P\ V4x)\ (ap\ V3f\ V4x)))))))) \quad (46)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27a^{A_27c}). (\forall V2x \in A_27c. ((ap (ap (ap (c_2Ecombin_2Eo\ A_27c\ A_27b\ A_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \quad (47)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap (c_2Ecombin_2EI\ A_27a)\ V0x) = V0x)) \quad (48)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0ll \in (ty_2Ellist_2Ellist\ A_27a). (((ap (c_2Ellist_2ELHD\ A_27a)\ V0ll) = (c_2Eoption_2ENONE\ A_27a)) \Leftrightarrow (V0ll = (c_2Ellist_2ELNIL\ A_27a))) \wedge (((c_2Eoption_2ENONE\ A_27a) = (ap (c_2Ellist_2ELHD\ A_27a)\ V0ll)) \Leftrightarrow (V0ll = (c_2Ellist_2ELNIL\ A_27a)))) \quad (49)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0h \in A_27a. (\forall V1t \in (ty_2Ellist_2Ellist\ A_27a). (\forall V2ll \in (ty_2Ellist_2Ellist\ A_27a). ((V2ll = (ap (ap (c_2Ellist_2ELCONS\ A_27a)\ V0h)\ V1t)) \Leftrightarrow ((ap (c_2Ellist_2ELHD\ A_27a)\ V2ll) = (ap (c_2Eoption_2ESOME\ A_27a)\ V0h)) \wedge ((ap (c_2Ellist_2ELTL\ A_27a)\ V2ll) = (ap (c_2Eoption_2ESOME\ (ty_2Ellist_2Ellist\ A_27a)\ V1t))))))) \quad (50)$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\
& \quad \forall V0f \in ((ty_2Eoption_2Eoption\ (ty_2Epair_2Eprod\ A_{.27a} \\
& \quad \quad A_{.27b}))^{A_{.27a}}).(\exists V1g \in ((ty_2Ellist_2Ellist\ A_{.27b})^{A_{.27a}}). \\
& \quad ((\forall V2x \in A_{.27a}.((ap\ (c_2Ellist_2ELHD\ A_{.27b})\ (ap\ V1g\ V2x)) = \\
& \quad (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ (ty_2Epair_2Eprod\ A_{.27a}\ A_{.27b}) \\
& \quad \quad A_{.27b})\ (c_2Epair_2ESND\ A_{.27a}\ A_{.27b}))\ (ap\ V0f\ V2x)))) \wedge (\forall V3x \in \\
& \quad A_{.27a}.((ap\ (c_2Ellist_2ELTL\ A_{.27b})\ (ap\ V1g\ V3x)) = (ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& \quad \quad (ty_2Epair_2Eprod\ A_{.27a}\ A_{.27b})\ (ty_2Ellist_2Ellist\ A_{.27b}))\ (\\
& \quad \quad ap\ (ap\ (c_2Ecombin_2Eo\ (ty_2Epair_2Eprod\ A_{.27a}\ A_{.27b})\ (ty_2Ellist_2Ellist \\
& \quad \quad \quad A_{.27b})\ A_{.27a})\ V1g)\ (c_2Epair_2EFST\ A_{.27a}\ A_{.27b})))\ (ap\ V0f\ V3x)))))) \\
& \hspace{15em} (51)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\
& (\forall V0v \in A_{.27b}.(\forall V1f \in (A_{.27b}^{A_{.27a}}).((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\
& \quad A_{.27a}\ A_{.27b})\ (c_2Eoption_2ENONE\ A_{.27a}))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\
& \quad A_{.27a}.(\forall V3v \in A_{.27b}.(\forall V4f \in (A_{.27b}^{A_{.27a}}).((ap\ (ap \\
& \quad (ap\ (c_2Eoption_2Eoption_CASE\ A_{.27a}\ A_{.27b})\ (ap\ (c_2Eoption_2ESOME \\
& \quad \quad A_{.27a})\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \\
& \hspace{15em} (52)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in \\
& \quad A_{.27a}.(((ap\ (c_2Eoption_2ESOME\ A_{.27a})\ V0x) = (ap\ (c_2Eoption_2ESOME \\
& \quad \quad A_{.27a})\ V1y))) \Leftrightarrow (V0x = V1y)))) \\
& \hspace{15em} (53)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\
& (\forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1x \in A_{.27a}.((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& \quad A_{.27a}\ A_{.27b})\ V0f)\ (ap\ (c_2Eoption_2ESOME\ A_{.27a})\ V1x)) = (ap\ (c_2Eoption_2ESOME \\
& \quad A_{.27b})\ (ap\ V0f\ V1x)))))) \wedge (\forall V2f \in (A_{.27b}^{A_{.27a}}).((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& \quad A_{.27a}\ A_{.27b})\ V2f)\ (c_2Eoption_2ENONE\ A_{.27a})) = (c_2Eoption_2ENONE \\
& \quad \quad A_{.27b})))) \\
& \hspace{15em} (54)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow (\\
& \quad \forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1x \in (ty_2Eoption_2Eoption \\
& \quad \quad A_{.27a}).(\forall V2y \in A_{.27b}.(((ap\ (ap\ (c_2Eoption_2EOPTION_MAP \\
& \quad \quad A_{.27a}\ A_{.27b})\ V0f)\ V1x) = (ap\ (c_2Eoption_2ESOME\ A_{.27b})\ V2y))) \Leftrightarrow (\exists V3z \in \\
& \quad \quad A_{.27a}.((V1x = (ap\ (c_2Eoption_2ESOME\ A_{.27a})\ V3z)) \wedge (V2y = (ap\ V0f \\
& \quad \quad \quad V3z)))))) \\
& \hspace{15em} (55)
\end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\ A_27a).((\exists V1x \in A_27a.(V0opt = (ap\ (c_2Eoption_2ESOME\ A_27a) \\ V1x))) \vee (V0opt = (c_2Eoption_2ENONE\ A_27a)))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0x \in A_27a.(\forall V1y \in A_27b.(\forall V2a \in A_27a.(\forall V3b \in \\ A_27b.(((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b))) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b).(\exists V1q \in A_27a. \\ (\exists V2r \in A_27b.(V0x = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b) \\ V1q)\ V2r)))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0x \in A_27a.(\forall V1y \in A_27b.((ap\ (c_2Epair_2EFST\ A_27a \\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0x \in A_27a.(\forall V1y \in A_27b.((ap\ (c_2Epair_2ESND\ A_27a \\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V1y))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}).(\forall V1x \in \\ A_27a.(\forall V2y \in A_27b.((ap\ (ap\ (c_2Epair_2EUNCURRY\ A_27a \\ A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y))) = \\ (ap\ (ap\ V0f\ V1x)\ V2y)))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c \Rightarrow (\forall V0x \in A_27b.(\forall V1y \in A_27c.(\forall V2f \in \\ ((A_27a^{A_27c})^{A_27b}).((ap\ (ap\ (c_2Epair_2Epair_2CASE\ A_27a\ A_27b \\ A_27c)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27b\ A_27c)\ V0x)\ V1y))\ V2f) = (ap \\ (ap\ V2f\ V0x)\ V1y)))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0a \in (ty_2Epath_2Epath\ A_27a\ A_27b).((ap\ (c_2Epath_2EtoPath \\ & A_27a\ A_27b)\ (ap\ (c_2Epath_2EfromPath\ A_27a\ A_27b)\ V0a)) = V0a)) \wedge \\ & (\forall V1r \in (ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod \\ & A_27b\ A_27a)))) .((ap\ (c_2Epath_2EfromPath\ A_27a\ A_27b)\ (ap\ (c_2Epath_2EtoPath \\ & A_27a\ A_27b)\ V1r)) = V1r))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0r \in (ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod \\ & A_27b\ A_27a)))) .(\forall V1r_27 \in (ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist \\ & (ty_2Epair_2Eprod\ A_27b\ A_27a)))) .(((ap\ (c_2Epath_2EtoPath\ A_27a \\ & A_27b)\ V0r) = (ap\ (c_2Epath_2EtoPath\ A_27a\ A_27b)\ V1r_27)) \Leftrightarrow (V0r = \\ & V1r_27))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in A_27a.(\forall V1y \in A_27a.(((ap\ (c_2Epath_2Estopped_at \\ & A_27a\ A_27b)\ V0x) = (ap\ (c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V1y)) \Leftrightarrow \\ & (V0x = V1y)))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0x \in A_27a.((ap\ (c_2Epath_2Efirst\ A_27a\ A_27b)\ (ap\ (c_2Epath_2Estopped_at \\ & A_27a\ A_27b)\ V0x)) = V0x)) \wedge (\forall V1x \in A_27a.(\forall V2r \in A_27b. \\ & (\forall V3p \in (ty_2Epath_2Epath\ A_27a\ A_27b).((ap\ (c_2Epath_2Efirst \\ & A_27a\ A_27b)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcns\ A_27a\ A_27b)\ V1x)\ V2r) \\ & V3p)) = V1x)))))) \end{aligned} \quad (66)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (67)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (68)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (69)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (70)$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (71)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (72)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (73)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (74)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (75)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (76)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow \forall A_27c. \\ & nonempty \ A_27c \Rightarrow (\forall V0f \in ((ty_2Epair_2Eprod \ A_27b \ (ty_2Eoption_2Eoption \\ & (ty_2Epair_2Eprod \ A_27c \ A_27a)))^{A_27a}).(\exists V1g \in ((ty_2Epath_2Epath \\ & A_27b \ A_27c)^{A_27a}).(\forall V2x \in A_27a.((ap \ V1g \ V2x) = (ap \ (ap \ (\\ & c_2Epair_2Epair_CASE \ (ty_2Epath_2Epath \ A_27b \ A_27c) \ A_27b \ (\\ & ty_2Eoption_2Eoption \ (ty_2Epair_2Eprod \ A_27c \ A_27a))) \ (ap \ V0f \\ & V2x)) \ (\lambda V3y \in A_27b.(\lambda V4v2 \in (ty_2Eoption_2Eoption \ (ty_2Epair_2Eprod \\ & A_27c \ A_27a)).(ap \ (ap \ (ap \ (c_2Eoption_2Eoption_CASE \ (ty_2Epair_2Eprod \\ & A_27c \ A_27a) \ (ty_2Epath_2Epath \ A_27b \ A_27c)) \ V4v2) \ (ap \ (c_2Epath_2Estopped_at \\ & A_27b \ A_27c) \ V3y)) \ (\lambda V5v3 \in (ty_2Epair_2Eprod \ A_27c \ A_27a). \\ & (ap \ (ap \ (c_2Epair_2Epair_CASE \ (ty_2Epath_2Epath \ A_27b \ A_27c) \\ & A_27c \ A_27a) \ V5v3) \ (\lambda V6l \in A_27c.(\lambda V7v \in A_27a.(ap \ (ap \ (ap \\ & (c_2Epath_2Epcns \ A_27b \ A_27c) \ V3y) \ V6l) \ (ap \ V1g \ V7v)))))))))) \end{aligned}$$