

thm_2Epath_2Epmmap__thm (TMaa- jwR6YsA99j7tfTYQU4ABGo3iGFTZDTd)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A.27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V0t \in 2.V0t)$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2.V2t)))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A0 A1) \tag{1}$$

Let `c_2Epair_2E_2FST` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a. \text{nonempty } A.27a \Rightarrow \forall A.27b. \text{nonempty } A.27b \Rightarrow \text{c_2Epair_2E_2FST } A.27a A.27b \in (A.27a (\text{ty_2Epair_2Eprod } A.27a A.27b)) \tag{2}$$

Let `ty_2Eone_2Eone` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Eone_2Eone} \tag{3}$$

Definition 7 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define `c_2Eone_2Eone` to be $(\text{ap } (\text{c_2Emin_2E_40 } \text{ty_2Eone_2Eone})) (\lambda V0x \in \text{ty_2Eone_2Eone})$

Definition 9 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t)) \text{c_2Ebool_2E_2F}))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (4)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (5)$$

Definition 10 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (6)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (7)$$

Definition 11 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (V0e))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (8)$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ellist_2Ellist\ A0) \quad (9)$$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_abs\ A_27a \in ((ty_2Ellist_2Ellist\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum}}) \quad (10)$$

Definition 12 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap\ (c_2Ellist_2Ellist_abs\ A_27a)\ (V0n))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (11)$$

Definition 13 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ V0x\ V1y)$

Let $ty_2Epath_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epath_2Epath\ A0\ A1) \quad (12)$$

Let $c_2Epath_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2EtoPath \\ & A_27a\ A_27b \in ((ty_2Epath_2Epath\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod\ A_27a\ A_27b))}) \end{aligned} \quad (13)$$

Definition 14 We define $c_2Epath_2Estopped_at$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. (ap\ (c_2Epath_2EtoPath\ A_27a\ A_27b)\ V0x)$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ & A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (14)$$

Let $c_2Epath_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2EfromPath \\ & A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)))^{(ty_2Epath_2Epath\ A_27a\ A_27b)}) \end{aligned} \quad (15)$$

Definition 15 We define $c_2Epath_2Efirst$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0p \in (ty_2Epath_2Epath\ A_27a\ A_27b)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (16)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (17)$$

Definition 16 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 17 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (18)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (19)$$

Definition 18 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ V0m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Definition 19 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Enum_2EZERO_REP)$

Definition 20 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (21)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2Ellist_rep\ A_27a \in \\ & (((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist\ A_27a)}) \end{aligned} \quad (22)$$

Definition 21 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS$

Definition 22 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_$

Definition 23 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 24 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota.\lambda V0h \in A_27a.\lambda V1t \in (ty_2Ellist_2Ellist\ A$

Definition 25 We define $c_2Epath_2Epcons$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1r \in A_27b.\lambda V2p$

Let $c_2Ellist_2ELMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Ellist_2ELMAP \\ & A_27a\ A_27b \in (((ty_2Ellist_2Ellist\ A_27b)^{(ty_2Ellist_2Ellist\ A_27a)})^{(A_27b^{A_27a})}) \end{aligned} \quad (23)$$

Definition 26 We define $c_2Epair_2E_23_23$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f \in (A_27$

Definition 27 We define c_2Epath_2Epmap to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f \in (A_27$

Assume the following.

$$True \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\ & True)) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0h1 \in A_27a.(\forall V1t1 \in \\ & (ty_2Ellist_2Ellist\ A_27a).(\forall V2h2 \in A_27a.(\forall V3t2 \in \\ & (ty_2Ellist_2Ellist\ A_27a).(((ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a) \\ & (V0h1)\ V1t1) = (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V2h2)\ V3t2)) \Leftrightarrow ((\\ & V0h1 = V2h2) \wedge (V1t1 = V3t2)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Ellist_2ELMAP\ A_27a\ A_27b) \\
& V0f)\ (c_2Ellist_2ELNIL\ A_27a)) = (c_2Ellist_2ELNIL\ A_27b))) \wedge \\
& (\forall V1f \in (A_27b^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in \\
& (ty_2Ellist_2Ellist\ A_27a).((ap\ (ap\ (c_2Ellist_2ELMAP\ A_27a \\
& A_27b)\ V1f)\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27a)\ V2h)\ V3t)) = (ap\ (ap \\
& (c_2Ellist_2ELCONS\ A_27b)\ (ap\ V1f\ V2h))\ (ap\ (ap\ (c_2Ellist_2ELMAP \\
& A_27a\ A_27b)\ V1f)\ V3t))))))))) \\
& \tag{28}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0x \in A_27a.(\forall V1y \in A_27b.(\forall V2a \in A_27a.(\forall V3b \in \\
& A_27b.(((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\
& (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))))) \\
& \tag{29}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0f \in (A_27b^{A_27a}). \\
& (\forall V1g \in (A_27d^{A_27c}).(\forall V2x \in A_27a.(\forall V3y \in \\
& A_27c.((ap\ (ap\ (ap\ (c_2Epair_2E_23_23\ A_27a\ A_27c\ A_27b\ A_27d) \\
& V0f)\ V1g)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27c)\ V2x)\ V3y)) = (ap\ (ap \\
& (c_2Epair_2E_2C\ A_27b\ A_27d)\ (ap\ V0f\ V2x))\ (ap\ V1g\ V3y))))))))) \\
& \tag{30}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\
& A_27a\ A_27b).(\forall V1f \in (A_27c^{A_27a}).(\forall V2g \in (A_27d^{A_27b}). \\
& ((ap\ (c_2Epair_2EFST\ A_27c\ A_27d)\ (ap\ (ap\ (ap\ (c_2Epair_2E_23_23 \\
& A_27a\ A_27b\ A_27c\ A_27d)\ V1f)\ V2g)\ V0p)) = (ap\ V1f\ (ap\ (c_2Epair_2EFST \\
& A_27a\ A_27b)\ V0p))))))))) \\
& \tag{31}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0p \in (ty_2Epair_2Eprod \\
& A_27a\ A_27b).(\forall V1f \in (A_27c^{A_27a}).(\forall V2g \in (A_27d^{A_27b}). \\
& ((ap\ (c_2Epair_2ESND\ A_27c\ A_27d)\ (ap\ (ap\ (ap\ (c_2Epair_2E_23_23 \\
& A_27a\ A_27b\ A_27c\ A_27d)\ V1f)\ V2g)\ V0p)) = (ap\ V2g\ (ap\ (c_2Epair_2ESND \\
& A_27a\ A_27b)\ V0p))))))))) \\
& \tag{32}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0a \in (ty_2Epath_2Epath\ A_27a\ A_27b).((ap\ (c_2Epath_2EtoPath \\
& A_27a\ A_27b)\ (ap\ (c_2Epath_2EfromPath\ A_27a\ A_27b)\ V0a)) = V0a)) \wedge \\
& (\forall V1r \in (ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod \\
& A_27b\ A_27a))))((ap\ (c_2Epath_2EfromPath\ A_27a\ A_27b)\ (ap\ (c_2Epath_2EtoPath \\
& A_27a\ A_27b)\ V1r)) = V1r))) \\
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0r \in (ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod \\
& A_27b\ A_27a))))(\forall V1r_27 \in (ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist \\
& (ty_2Epair_2Eprod\ A_27b\ A_27a))))((ap\ (c_2Epath_2EtoPath\ A_27a \\
& A_27b)\ V0r) = (ap\ (c_2Epath_2EtoPath\ A_27a\ A_27b)\ V1r_27)) \Leftrightarrow (V0r = \\
& V1r_27)) \\
\end{aligned} \tag{34}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0f \in (A_27b^{A_27a}). \\
& (\forall V1g \in (A_27c^{A_27d}).((\forall V2x \in A_27a.((ap\ (ap\ (\\
& c_2Epath_2Epmap\ A_27a\ A_27d\ A_27b\ A_27c)\ V0f)\ V1g)\ (ap\ (c_2Epath_2Estopped_at \\
& A_27a\ A_27d)\ V2x)) = (ap\ (c_2Epath_2Estopped_at\ A_27b\ A_27c)\ (\\
& ap\ V0f\ V2x)))) \wedge (\forall V3x \in A_27a.(\forall V4r \in A_27d.(\forall V5p \in \\
& (ty_2Epath_2Epath\ A_27a\ A_27d).((ap\ (ap\ (ap\ (c_2Epath_2Epmap \\
& A_27a\ A_27d\ A_27b\ A_27c)\ V0f)\ V1g)\ (ap\ (ap\ (ap\ (c_2Epath_2Epmap \\
& A_27a\ A_27d)\ V3x)\ V4r)\ V5p)) = (ap\ (ap\ (ap\ (c_2Epath_2Epmap\ A_27b \\
& A_27c)\ (ap\ V0f\ V3x))\ (ap\ V1g\ V4r))\ (ap\ (ap\ (ap\ (c_2Epath_2Epmap\ A_27a \\
& A_27d\ A_27b\ A_27c)\ V0f)\ V1g)\ V5p))))))))))
\end{aligned}$$