

thm_2Epath_2Estopped_at_not_pcons (TMVttGMKHWQj3x8bnvBmHgwEiArXgXSWr2k)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 9 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{3}$$

Definition 10 We define c_Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_Esum_2EABS$
Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (5)$$

Definition 11 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a) ($

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty ty_2Eenum_2Eenum \quad (6)$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \quad (7)$$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ellist_2Ellist_abs A_27a \in ((ty_2Ellist_2Ellist A_27a)^{(ty_2Eoption_2Eoption A_27a)^{ty_2Eenum_2Eenum}}) \quad (8)$$

Definition 12 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap (c_2Ellist_2Ellist_abs A_27a) (\lambda V0n \in ty$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (9)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (10)$$

Definition 13 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2$

Let $ty_2Epath_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Epath_2Epath A0 A1) \quad (11)$$

Let $c_2Epath_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epath_2EtoPath A_27a A_27b \in ((ty_2Epath_2Epath A_27a A_27b)^{(ty_2Epair_2Eprod A_27a (ty_2Ellist_2Ellist (ty_2Epair_2Eprod$$

(12)

Definition 14 We define $c_Epath_Estopped_at$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. (ap (c_Epath$

Let $c_Epath_EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epath_EfromPath \\ & A_27a A_27b \in ((ty_Epair_Eprod A_27a (ty_Elist_Elist (ty_Epair_Eprod \\ & \quad A_27b A_27a)))^{(ty_Epath_Epath A_27a A_27b)}) \end{aligned} \quad (13)$$

Let $c_Epair_EESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_EESND \\ & A_27a A_27b \in (A_27b^{(ty_Epair_Eprod A_27a A_27b)}) \end{aligned} \quad (14)$$

Let $c_Epair_EFEFSST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_EFEFSST \\ & A_27a A_27b \in (A_27a^{(ty_Epair_Eprod A_27a A_27b)}) \end{aligned} \quad (15)$$

Definition 15 We define c_Epath_Efirst to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0p \in (ty_Epath_Epath A_27a A_27b)$

Let $c_Eenum_EZERO_REP : \iota$ be given. Assume the following.

$$c_Eenum_EZERO_REP \in \omega \quad (16)$$

Let $c_Eenum_EABS_num : \iota$ be given. Assume the following.

$$c_Eenum_EABS_num \in (ty_Eenum_Eenum^{\omega}) \quad (17)$$

Definition 16 We define c_Eenum_E0 to be $(ap c_Eenum_EABS_num c_Eenum_EZERO_REP)$.

Definition 17 We define $c_Earithmetic_EZERO$ to be c_Eenum_E0 .

Let $c_Eenum_EREP_num : \iota$ be given. Assume the following.

$$c_Eenum_EREP_num \in (\omega^{ty_Eenum_Eenum}) \quad (18)$$

Let $c_Eenum_ESUC_REP : \iota$ be given. Assume the following.

$$c_Eenum_ESUC_REP \in (\omega^{\omega}) \quad (19)$$

Definition 18 We define c_Eenum_ESUC to be $\lambda V0m \in ty_Eenum_Eenum. (ap c_Eenum_EABS_num$

Let $c_Earithmetic_E2B : \iota$ be given. Assume the following.

$$c_Earithmetic_E2B \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum}) \quad (20)$$

Definition 19 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_Eenum_Eenum. (ap (ap c_Earithmetic$

Definition 20 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_Eenum_Eenum. V0x$.

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ & \quad A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ & \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\ & \hspace{15em} (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0r \in (ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod \\ & \quad A_27b\ A_27a))). (\forall V1r_27 \in (ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist \\ & \quad (ty_2Epair_2Eprod\ A_27b\ A_27a))). (((ap\ (c_2Epath_2EtoPath\ A_27a \\ & \quad A_27b)\ V0r) = (ap\ (c_2Epath_2EtoPath\ A_27a\ A_27b)\ V1r_27)) \Leftrightarrow (V0r = \\ & \quad V1r_27))) \\ & \hspace{15em} (31) \end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27a. (\forall V2r \in A_27b. (\forall V3p \in \\ & \quad (ty_2Epath_2Epath\ A_27a\ A_27b). (\neg((ap\ (c_2Epath_2Estopped_at \\ & \quad A_27a\ A_27b)\ V0x) = (ap\ (ap\ (ap\ (c_2Epath_2Econs\ A_27a\ A_27b)\ V1y) \\ & \quad V2r)\ V3p))) \wedge (\neg((ap\ (ap\ (ap\ (c_2Epath_2Econs\ A_27a\ A_27b)\ V1y) \\ & \quad V2r)\ V3p) = (ap\ (c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V0x)))))) \\ & \hspace{15em} \end{aligned}$$