

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (6)$$

Definition 8 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Definition 9 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (7)$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \quad (8)$$

Let $c_2Ellist_2Ellist_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ellist_2Ellist_rep A_27a \in ((ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum})^{(ty_2Ellist_2Ellist A_27a)} \quad (9)$$

Let $c_2Ellist_2Ellist_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ellist_2Ellist_abs A_27a \in ((ty_2Ellist_2Ellist A_27a)^{(ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum}}) \quad (10)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (11)$$

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.t1 t2))))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (12)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (13)$$

Definition 12 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS_sum))$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (14)$$

Definition 13 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Definition 14 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone.\ V0x))$

Definition 15 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 16 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$

Definition 17 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS\ A_27a)\ V0e)$

Definition 18 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ 0)$

Definition 19 We define $c_2Ellist_2ELHD$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0ll))$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2Eoption_CASE\ A_27a\ A_27b \in (((A_27b^{(A_27b^{A_27a}})^{A_27b})^{(ty_2Eoption_2Eoption\ A_27a)})^{A_27b}) \quad (15)$$

Definition 20 We define $c_2Ellist_2ELTL$ to be $\lambda A_27a : \iota. \lambda V0ll \in (ty_2Ellist_2Ellist\ A_27a). (ap\ (ap\ (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0ll)\ c_2Ellist_2ELTL))$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_MAP\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27b)^{(ty_2Eoption_2Eoption\ A_27a)})^{(A_27b^{A_27a}})^{A_27b}) \quad (16)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (17)$$

Let $c_2Ellist_2ELTAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ellist_2ELTAKE\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Ellist_2Ellist\ A_27a)})^{ty_2Enum_2Enum} \quad (18)$$

Let $c_2Eoption_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2ETHE\ A_27a \in (A_27a^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (19)$$

Definition 21 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ V0P)\ c_2Ebool_2E_3F)))$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (20)$$

Definition 22 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 23 We define $c_2Ellist_2ELCONS$ to be $\lambda A_27a : \iota. \lambda V0h \in A_27a. \lambda V1t \in (ty_2Ellist_2Ellist A_27a$

Definition 24 We define $c_2Ellist_2ELNIL$ to be $\lambda A_27a : \iota. (ap (c_2Ellist_2Ellist_abs A_27a) (\lambda V0n \in ty$

Let $ty_2Epath_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epath_2Epath A0 A1) \quad (21)$$

Let $c_2Epath_2Elabels : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epath_2Elabels A_27a A_27b \in ((ty_2Ellist_2Ellist A_27b)^{(ty_2Epath_2Epath A_27a A_27b)}) \quad (22)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (23)$$

Let $c_2Epath_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epath_2EfromPath A_27a A_27b \in ((ty_2Epair_2Eprod A_27a (ty_2Ellist_2Ellist (ty_2Epair_2Eprod A_27b A_27a)))^{(ty_2Epath_2Epath A_27a A_27b)}) \quad (24)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (25)$$

Definition 25 We define $c_2Epath_2Efirst$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0p \in (ty_2Epath_2Epath A_27a A_27b$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (26)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (27)$$

Definition 26 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2$

Let $c_2Epath_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2EtoPath \\ & A_27a\ A_27b \in ((ty_2Epath_2Epath\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ (ty_2Ellist_2Ellist\ (ty_2Epair_2Eprod\ A_27a\ A_27b))})} \\ & \hspace{15em} (28) \end{aligned}$$

Definition 27 We define c_2Epath_2Epcns to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1r \in A_27b.\lambda V2p \in A_27c$

Definition 28 We define $c_2Epath_2Estopped_at$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.(ap\ (c_2Epath_2Epcns\ A_27a\ A_27b)\ V0x)$

Definition 29 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 30 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a \times A_27b})$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ & A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \\ & \hspace{15em} (29) \end{aligned}$$

Definition 31 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EIN)$.

Definition 32 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ V1t2)\ V0t1))$

Definition 33 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27a)\ V1t)\ V0s$

Definition 34 We define $c_2Epath_2Eokpath_f$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in (((2^{A_27a})^{A_27b})^{A_27a})$.

Definition 35 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27a)\ V1t)\ V0s$

Definition 36 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27a)\ V0P)$

Definition 37 We define $c_2EfixedPoint_2Egfp$ to be $\lambda A_27a : \iota.\lambda V0f \in ((2^{A_27a})^{(2^{A_27a})}).(ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27a)\ V0f)$

Definition 38 We define $c_2Epath_2Eokpath$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in (((2^{A_27a})^{A_27b})^{A_27a})$.

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \\ & \hspace{15em} (30) \end{aligned}$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \\ & \hspace{15em} (31) \end{aligned}$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \\ & \hspace{15em} (32) \end{aligned}$$

Definition 39 We define $c_2Epath_2Etrace_machine$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}).\lambda V$

Assume the following.

$$True \quad (33)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (37)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A_27a.(p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a.(p\ (ap\ V1Q\ V4x))))))) \quad (42)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((p\ V0P) \wedge (\forall V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \quad (43)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. ((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3)))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))) \quad (44)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow ((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (45)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0l \in (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ (c_2Elist_2ENIL\ A_27a))\ V0l) = V0l)) \wedge (\forall V1l1 \in (ty_2Elist_2Elist\ A_27a). (\forall V2l2 \in (ty_2Elist_2Elist\ A_27a). (\forall V3h \in A_27a. ((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3h)\ V1l1))\ V2l2) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3h)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V1l1)\ V2l2))))))) \quad (46)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0l)\ (c_2Elist_2ENIL\ A_27a)) = V0l)) \quad (47)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist\ A_27a). (\forall V1l2 \in (ty_2Elist_2Elist\ A_27a). (\forall V2l3 \in (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0l1)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V1l2)\ V2l3)) = (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V0l1)\ V1l2))\ V2l3)))))) \quad (48)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow ((ap\ (c_2Elist_2ELHD\ A_27a)\ (c_2Elist_2ELNIL\ A_27a)) = (c_2Eoption_2ENONE\ A_27a)) \wedge (\forall V0h \in A_27b. (\forall V1t \in (ty_2Elist_2Elist\ A_27b). ((ap\ (c_2Elist_2ELHD\ A_27b)\ (ap\ (ap\ (c_2Elist_2ELCONS\ A_27b)\ V0h)\ V1t)) = (ap\ (c_2Eoption_2ESOME\ A_27b)\ V0h)))) \quad (49)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& ((ap\ (c_2Ellist_2ELTL\ A_27a)\ (c_2Ellist_2ELNIL\ A_27a)) = (c_2Eoption_2ENONE \\
& \quad (ty_2Ellist_2Ellist\ A_27a))) \wedge (\forall V0h \in A_27b. (\forall V1t \in \\
& \quad (ty_2Ellist_2Ellist\ A_27b). ((ap\ (c_2Ellist_2ELTL\ A_27b)\ (ap \\
& \quad (ap\ (c_2Ellist_2ELCONS\ A_27b)\ V0h)\ V1t)) = (ap\ (c_2Eoption_2ESOME \\
& \quad (ty_2Ellist_2Ellist\ A_27b)\ V1t))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0ll \in (ty_2Ellist_2Ellist \\
& \quad A_27a). ((ap\ (ap\ (c_2Ellist_2ELTAKE\ A_27a)\ c_2Enum_2E0)\ V0ll) = \\
& \quad (ap\ (c_2Eoption_2ESOME\ (ty_2Elist_2Elist\ A_27a))\ (c_2Elist_2ENIL \\
& \quad A_27a)))) \wedge (\forall V1n \in ty_2Enum_2Enum. (\forall V2ll \in (ty_2Ellist_2Ellist \\
& \quad A_27a). ((ap\ (ap\ (c_2Ellist_2ELTAKE\ A_27a)\ (ap\ c_2Enum_2ESUC\ V1n)) \\
& \quad V2ll) = (ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ (ty_2Eoption_2Eoption \\
& \quad (ty_2Elist_2Elist\ A_27a)))\ (ap\ (c_2Ellist_2ELHD\ A_27a)\ V2ll)) \\
& \quad (c_2Eoption_2ENONE\ (ty_2Elist_2Elist\ A_27a)))\ (\lambda V3hd \in A_27a. \\
& \quad (ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE\ (ty_2Elist_2Elist\ A_27a) \\
& \quad (ty_2Eoption_2Eoption\ (ty_2Elist_2Elist\ A_27a)))\ (ap\ (ap\ (c_2Ellist_2ELTAKE \\
& \quad A_27a)\ V1n)\ (ap\ (c_2Eoption_2ETHE\ (ty_2Ellist_2Ellist\ A_27a)) \\
& \quad (ap\ (c_2Ellist_2ELTL\ A_27a)\ V2ll))))\ (c_2Eoption_2ENONE\ (ty_2Elist_2Elist \\
& \quad A_27a)))\ (\lambda V4tl \in (ty_2Elist_2Elist\ A_27a). (ap\ (c_2Eoption_2ESOME \\
& \quad (ty_2Elist_2Elist\ A_27a))\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3hd) \\
& \quad V4tl))))))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow ((\forall V0l \in (ty_2Ellist_2Ellist\ A_27a). ((\\
& \quad ap\ (ap\ (c_2Ellist_2ELTAKE\ A_27a)\ c_2Enum_2E0)\ V0l) = (ap\ (c_2Eoption_2ESOME \\
& \quad (ty_2Elist_2Elist\ A_27a))\ (c_2Elist_2ENIL\ A_27a)))) \wedge ((\forall V1n \in \\
& \quad ty_2Enum_2Enum. ((ap\ (ap\ (c_2Ellist_2ELTAKE\ A_27b)\ (ap\ c_2Enum_2ESUC \\
& \quad V1n))\ (c_2Ellist_2ELNIL\ A_27b)) = (c_2Eoption_2ENONE\ (ty_2Elist_2Elist \\
& \quad A_27b)))) \wedge (\forall V2n \in ty_2Enum_2Enum. (\forall V3h \in A_27c. \\
& \quad (\forall V4t \in (ty_2Ellist_2Ellist\ A_27c). ((ap\ (ap\ (c_2Ellist_2ELTAKE \\
& \quad A_27c)\ (ap\ c_2Enum_2ESUC\ V2n))\ (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27c) \\
& \quad V3h)\ V4t)) = (ap\ (ap\ (c_2Eoption_2EOPTION_MAP\ (ty_2Elist_2Elist \\
& \quad A_27c)\ (ty_2Elist_2Elist\ A_27c))\ (ap\ (c_2Elist_2ECONS\ A_27c) \\
& \quad V3h))\ (ap\ (ap\ (c_2Ellist_2ELTAKE\ A_27c)\ V2n)\ V4t))))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2ty_2Enum_2Enum). (((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge \\
& (\forall V1n \in ty_2Enum_2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC \\
& \quad V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. (p\ (ap\ V0P\ V2n))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\ A_27a).((V0opt = (c_2Eoption_2ENONE\ A_27a)) \vee (\exists V1x \in A_27a. \\ (V0opt = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ (\forall V0v \in A_27b.(\forall V1f \in (A_27b^{A_27a}).((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\ A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\ A_27a.(\forall V3v \in A_27b.(\forall V4f \in (A_27b^{A_27a}).((ap\ (ap \\ (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME \\ A_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x))))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ A_27a.(((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\ A_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\neg((c_2Eoption_2ENONE \\ A_27a) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x)))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((ap\ (c_2Eoption_2ETHE \\ A_27a)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x)) = V0x)) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0p \in (ty_2Epath_2Epath\ A_27a\ A_27b).((\exists V1x \in A_27a. \\ (V0p = (ap\ (c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V1x))) \vee (\exists V2x \in \\ A_27a.(\exists V3r \in A_27b.(\exists V4q \in (ty_2Epath_2Epath\ A_27a \\ A_27b).(V0p = (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A_27a\ A_27b)\ V2x)\ V3r) \\ V4q))))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ (\forall V0x \in A_27a.((ap\ (c_2Epath_2Efirst\ A_27a\ A_27b)\ (ap\ (c_2Epath_2Estopped_at \\ A_27a\ A_27b)\ V0x)) = V0x)) \wedge (\forall V1x \in A_27a.(\forall V2r \in A_27b. \\ (\forall V3p \in (ty_2Epath_2Epath\ A_27a\ A_27b).((ap\ (c_2Epath_2Efirst \\ A_27a\ A_27b)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcons\ A_27a\ A_27b)\ V1x)\ V2r) \\ V3p)) = V1x)))))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0x \in A_27a.((ap\ (c_2Epath_2Elabels\ A_27a\ A_27b)\ (ap\ (\\ & c_2Epath_2Estopped_at\ A_27a\ A_27b)\ V0x)) = (c_2Ellist_2ELNIL \\ & A_27b))) \wedge (\forall V1x \in A_27a.(\forall V2r \in A_27b.(\forall V3p \in \\ & (ty_2Epath_2Epath\ A_27a\ A_27b).((ap\ (c_2Epath_2Elabels\ A_27a \\ & A_27b)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcns\ A_27a\ A_27b)\ V1x)\ V2r)\ V3p)) = \\ & (ap\ (ap\ (c_2Ellist_2ELCONS\ A_27b)\ V2r)\ (ap\ (c_2Epath_2Elabels \\ & A_27a\ A_27b)\ V3p)))))) \\ & \end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0R \in (((2^{A_27a})^{A_27b})^{A_27a}).((\forall V1x \in A_27a.(\\ & p\ (ap\ (ap\ (c_2Epath_2Eokpath\ A_27a\ A_27b)\ V0R)\ (ap\ (c_2Epath_2Estopped_at \\ & A_27a\ A_27b)\ V1x)))) \wedge (\forall V2x \in A_27a.(\forall V3r \in A_27b. \\ & (\forall V4p \in (ty_2Epath_2Epath\ A_27a\ A_27b).((p\ (ap\ (ap\ (c_2Epath_2Eokpath \\ & A_27a\ A_27b)\ V0R)\ (ap\ (ap\ (ap\ (c_2Epath_2Epcns\ A_27a\ A_27b)\ V2x) \\ & V3r)\ V4p))) \Leftrightarrow ((p\ (ap\ (ap\ (ap\ V0R\ V2x)\ V3r)\ (ap\ (c_2Epath_2Efirst\ A_27a \\ & A_27b)\ V4p))) \wedge (p\ (ap\ (ap\ (c_2Epath_2Eokpath\ A_27a\ A_27b)\ V0R)\ V4p)))))) \\ & \end{aligned} \tag{62}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{63}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{64}$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\ & \end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\ & \end{aligned} \tag{66}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{67}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))) \\ & \end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \wedge (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (\neg(p \vee V1q)) \vee \neg(p \vee V2r)))) \wedge (((p \vee V1q) \vee \\
& (\neg(p \vee V0p))) \wedge ((p \vee V2r) \vee \neg(p \vee V0p))))))))) \\
& \hspace{15em} (69)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \vee (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee \neg(p \vee V1q)) \wedge ((p \vee V0p) \vee \neg(p \vee V2r))) \wedge \\
& ((p \vee V1q) \vee ((p \vee V2r) \vee \neg(p \vee V0p))))))))) \\
& \hspace{15em} (70)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \Rightarrow (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (p \vee V1q)) \wedge ((p \vee V0p) \vee \neg(p \vee V2r))) \wedge (\\
& \neg(p \vee V1q) \vee ((p \vee V2r) \vee \neg(p \vee V0p))))))))) \\
& \hspace{15em} (71)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee V0p) \Leftrightarrow \neg(p \vee V1q)) \Leftrightarrow (((p \vee V0p) \vee \\
& (p \vee V1q)) \wedge (\neg(p \vee V1q) \vee \neg(p \vee V0p)))))) \\
& \hspace{15em} (72)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow (p \vee V0p)))) \\
& \hspace{15em} (73)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \Rightarrow (p \vee V1q))) \Rightarrow \neg(p \vee V1q)))) \\
& \hspace{15em} (74)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow \neg(p \vee V0p)))) \\
& \hspace{15em} (75)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \vee V0p) \vee (p \vee V1q))) \Rightarrow \neg(p \vee V1q)))) \\
& \hspace{15em} (76)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. ((\neg(\neg(p \vee V0p))) \Rightarrow (p \vee V0p))) \\
& \hspace{15em} (77)
\end{aligned}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0n \in ty_2Enum_2Enum. (\forall V1l \in (ty_2Elist_2Elist \\ & \quad A_27a). (\forall V2P \in (2^{(ty_2Elist_2Elist\ A_27a)}). (\forall V3p \in \\ & \quad (ty_2Epath_2Epath\ (ty_2Elist_2Elist\ A_27a)\ A_27a). (\forall V4init \in \\ & \quad A_27b. (((p\ (ap\ (ap\ (c_2Epath_2Eokpath\ (ty_2Elist_2Elist\ A_27a) \\ & \quad A_27a)\ (ap\ (c_2Epath_2Etrace_machine\ A_27a)\ V2P))\ V3p)) \wedge (p\ (\\ & \quad ap\ V2P\ (ap\ (c_2Epath_2Efirst\ (ty_2Elist_2Elist\ A_27a)\ A_27a)\ V3p)))) \Rightarrow \\ & \quad (((ap\ (ap\ (c_2Elist_2ELTAKE\ A_27a)\ V0n)\ (ap\ (c_2Epath_2Elabels \\ & \quad (ty_2Elist_2Elist\ A_27a)\ A_27a)\ V3p)) = (ap\ (c_2Eoption_2ESOME \\ & \quad (ty_2Elist_2Elist\ A_27a)\ V1l)) \Rightarrow (p\ (ap\ V2P\ (ap\ (ap\ (c_2Elist_2EAPPEND \\ & \quad A_27a)\ (ap\ (c_2Epath_2Efirst\ (ty_2Elist_2Elist\ A_27a)\ A_27a) \\ & \quad V3p))\ V1l)))))))))) \end{aligned}$$