

thm\_2Epath\_2Eunfold\_\_thm2  
(TMLFwX4mV4c4sN3ELYvtL3jxvAT3ispVkn3)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 5** We define  $c\_2Ebool\_2E\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2BOUNDED$  to be  $(\lambda V0v \in 2.c\_2Ebool\_2E\_2ET)$ .

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty \ ty\_2Eone\_2Eone \tag{1}$$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow \forall A1.nonempty \ A1 \Rightarrow nonempty \ (ty\_2Esum\_2Esum \ A0 \ A1) \tag{2}$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum \ A\_27a \ A\_27b \in ((ty\_2Esum\_2Esum \ A\_27a \ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \tag{3}$$

**Definition 8** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap (c\_2Esum\_2EABS\_sum$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (4)$$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (5)$$

**Definition 9** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_ABS\ A\_27a)\ V0x)$

**Definition 10** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge P\ x)) \text{ of type } \iota \Rightarrow \iota.$

**Definition 11** We define  $c\_2Eone\_2Eone$  to be  $(ap\ (c\_2Emin\_2E\_40\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2Eone.\ V0x))$

**Definition 12** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_7E))$

**Definition 13** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS\ A\_27a\ A\_27b)\ V0e)$

**Definition 14** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_ABS\ A\_27a)\ (c\_2Eone\_2Eone\ A\_27a))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (6)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (7)$$

**Definition 15** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ V0x\ V1y)$

Let  $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \quad (8)$$

Let  $ty\_2Epath\_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epath\_2Epath\ A0\ A1) \quad (9)$$

Let  $c\_2Epath\_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2EfromPath\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Ellist\_2Ellist\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)))^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}) \quad (10)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (11)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (12)$$

**Definition 16** We define  $c\_2Epath\_2Efirst$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b)$ .

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (13)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (14)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (15)$$

**Definition 17** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 18** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (16)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (17)$$

**Definition 19** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num\ m)$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (18)$$

**Definition 20** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ n))$ .

**Definition 21** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (19)$$

Let  $c\_2Ellist\_2Ellist\_rep : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_rep\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)} \quad (20)$$

**Definition 22** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

Let  $c\_2Ellist\_2Ellist\_abs : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_abs\ A\_27a \in ((ty\_2Ellist\_2Ellist\ A\_27a)^{(ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum}}) \quad (21)$$

**Definition 23** We define  $c\_2Ellist\_2ELCONS$  to be  $\lambda A\_27a : \iota. \lambda V0h \in A\_27a. \lambda V1t \in (ty\_2Ellist\_2Ellist\ A\_27a.$

Let  $c\_2Epath\_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2EtoPath\ A\_27a\ A\_27b \in ((ty\_2Epath\_2Epath\ A\_27a\ A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Ellist\_2Ellist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))}) \quad (22)$$

**Definition 24** We define  $c\_2Epath\_2Epcns$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1r \in A\_27b. \lambda V2p \in$

**Definition 25** We define  $c\_2Epair\_2Epair\_CASE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0p \in (ty\_2Epair\_2Epair\ A\_27a\ A\_27b\ A\_27c.$

**Definition 26** We define  $c\_2Ellist\_2ELNIL$  to be  $\lambda A\_27a : \iota. (ap\ (c\_2Ellist\_2Ellist\_abs\ A\_27a)\ (\lambda V0n \in ty\_2Ellist\_2Ellist\ A\_27a.$

**Definition 27** We define  $c\_2Epath\_2Estopped\_at$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. (ap\ (c\_2Epath\_2EtoPath\ A\_27a\ A\_27b.$

Let  $c\_2Eoption\_2Eoption\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (23)$$

**Definition 28** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in ((A\_27c^{A\_27a})^{(ty\_2Epair\_2Epair\ A\_27a\ A\_27b.$

Let  $c\_2Eoption\_2EOPTION\_MAP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_MAP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \quad (24)$$

**Definition 29** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V0x \in A\_27b.$

**Definition 30** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in (A\_27b^{A\_27c}). \lambda V1g \in (A\_27c^{A\_27a}). \lambda V2h \in (A\_27a^{A\_27b}). \lambda V3i \in (A\_27b^{A\_27c}). \lambda V4j \in (A\_27c^{A\_27a}). \lambda V5k \in (A\_27a^{A\_27b}). \lambda V6l \in (A\_27b^{A\_27c}). \lambda V7m \in (A\_27c^{A\_27a}). \lambda V8n \in (A\_27a^{A\_27b}). \lambda V9o \in (A\_27b^{A\_27c}). \lambda V10p \in (A\_27c^{A\_27a}). \lambda V11q \in (A\_27a^{A\_27b}). \lambda V12r \in (A\_27b^{A\_27c}). \lambda V13s \in (A\_27c^{A\_27a}). \lambda V14t \in (A\_27a^{A\_27b}). \lambda V15u \in (A\_27b^{A\_27c}). \lambda V16v \in (A\_27c^{A\_27a}). \lambda V17w \in (A\_27a^{A\_27b}). \lambda V18x \in (A\_27b^{A\_27c}). \lambda V19y \in (A\_27c^{A\_27a}). \lambda V20z \in (A\_27a^{A\_27b}). \lambda V21aa \in (A\_27b^{A\_27c}). \lambda V22ab \in (A\_27c^{A\_27a}). \lambda V23ba \in (A\_27a^{A\_27b}). \lambda V24bb \in (A\_27b^{A\_27c}). \lambda V25ca \in (A\_27c^{A\_27a}). \lambda V26cb \in (A\_27a^{A\_27b}). \lambda V27cb \in (A\_27b^{A\_27c}). \lambda V28cc \in (A\_27c^{A\_27a}). \lambda V29ca \in (A\_27a^{A\_27b}). \lambda V30ca \in (A\_27b^{A\_27c}). \lambda V31ca \in (A\_27c^{A\_27a}). \lambda V32ca \in (A\_27a^{A\_27b}). \lambda V33ca \in (A\_27b^{A\_27c}). \lambda V34ca \in (A\_27c^{A\_27a}). \lambda V35ca \in (A\_27a^{A\_27b}). \lambda V36ca \in (A\_27b^{A\_27c}). \lambda V37ca \in (A\_27c^{A\_27a}). \lambda V38ca \in (A\_27a^{A\_27b}). \lambda V39ca \in (A\_27b^{A\_27c}). \lambda V40ca \in (A\_27c^{A\_27a}). \lambda V41ca \in (A\_27a^{A\_27b}). \lambda V42ca \in (A\_27b^{A\_27c}). \lambda V43ca \in (A\_27c^{A\_27a}). \lambda V44ca \in (A\_27a^{A\_27b}). \lambda V45ca \in (A\_27b^{A\_27c}). \lambda V46ca \in (A\_27c^{A\_27a}). \lambda V47ca \in (A\_27a^{A\_27b}). \lambda V48ca \in (A\_27b^{A\_27c}). \lambda V49ca \in (A\_27c^{A\_27a}). \lambda V50ca \in (A\_27a^{A\_27b}). \lambda V51ca \in (A\_27b^{A\_27c}). \lambda V52ca \in (A\_27c^{A\_27a}). \lambda V53ca \in (A\_27a^{A\_27b}). \lambda V54ca \in (A\_27b^{A\_27c}). \lambda V55ca \in (A\_27c^{A\_27a}). \lambda V56ca \in (A\_27a^{A\_27b}). \lambda V57ca \in (A\_27b^{A\_27c}). \lambda V58ca \in (A\_27c^{A\_27a}). \lambda V59ca \in (A\_27a^{A\_27b}). \lambda V60ca \in (A\_27b^{A\_27c}). \lambda V61ca \in (A\_27c^{A\_27a}). \lambda V62ca \in (A\_27a^{A\_27b}). \lambda V63ca \in (A\_27b^{A\_27c}). \lambda V64ca \in (A\_27c^{A\_27a}). \lambda V65ca \in (A\_27a^{A\_27b}). \lambda V66ca \in (A\_27b^{A\_27c}). \lambda V67ca \in (A\_27c^{A\_27a}). \lambda V68ca \in (A\_27a^{A\_27b}). \lambda V69ca \in (A\_27b^{A\_27c}). \lambda V70ca \in (A\_27c^{A\_27a}). \lambda V71ca \in (A\_27a^{A\_27b}). \lambda V72ca \in (A\_27b^{A\_27c}). \lambda V73ca \in (A\_27c^{A\_27a}). \lambda V74ca \in (A\_27a^{A\_27b}). \lambda V75ca \in (A\_27b^{A\_27c}). \lambda V76ca \in (A\_27c^{A\_27a}). \lambda V77ca \in (A\_27a^{A\_27b}). \lambda V78ca \in (A\_27b^{A\_27c}). \lambda V79ca \in (A\_27c^{A\_27a}). \lambda V80ca \in (A\_27a^{A\_27b}). \lambda V81ca \in (A\_27b^{A\_27c}). \lambda V82ca \in (A\_27c^{A\_27a}). \lambda V83ca \in (A\_27a^{A\_27b}). \lambda V84ca \in (A\_27b^{A\_27c}). \lambda V85ca \in (A\_27c^{A\_27a}). \lambda V86ca \in (A\_27a^{A\_27b}). \lambda V87ca \in (A\_27b^{A\_27c}). \lambda V88ca \in (A\_27c^{A\_27a}). \lambda V89ca \in (A\_27a^{A\_27b}). \lambda V90ca \in (A\_27b^{A\_27c}). \lambda V91ca \in (A\_27c^{A\_27a}). \lambda V92ca \in (A\_27a^{A\_27b}). \lambda V93ca \in (A\_27b^{A\_27c}). \lambda V94ca \in (A\_27c^{A\_27a}). \lambda V95ca \in (A\_27a^{A\_27b}). \lambda V96ca \in (A\_27b^{A\_27c}). \lambda V97ca \in (A\_27c^{A\_27a}). \lambda V98ca \in (A\_27a^{A\_27b}). \lambda V99ca \in (A\_27b^{A\_27c}). \lambda V100ca \in (A\_27c^{A\_27a}).$

Let  $c\_2Eoption\_2EOPTION\_BIND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_BIND \\ & A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}})^{(ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}}) \end{aligned} \quad (25)$$

Let  $c\_2Earithmetic\_2EFUNPOW : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Earithmetic\_2EFUNPOW\ A\_27a \in \\ & (((A\_27a^{A\_27a})^{ty\_2Enum\_2Enum})^{(A\_27a)^{A\_27a}}) \end{aligned} \quad (26)$$

**Definition 31** We define  $c\_2Ellist\_2ELUNFOLD$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((ty\_2Eoption\_2Eoption$

**Definition 32** We define  $c\_2Epath\_2Eunfold$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0proj \in (A\_27a^{A\_27c})$

Assume the following.

$$True \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\ & True)) \end{aligned} \quad (29)$$

Assume the following.

$$(\forall V0v \in 2.((p\ (ap\ c\_2Ebool\_2EBOUNDED\ V0v)) \Leftrightarrow True)) \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & (\forall V0v \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_CASE \\ & A\_27a\ A\_27b)\ (c\_2Eoption\_2ENONE\ A\_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\ & A\_27a.(\forall V3v \in A\_27b.(\forall V4f \in (A\_27b^{A\_27a}).((ap\ (ap \\ & (ap\ (c\_2Eoption\_2Eoption\_CASE\ A\_27a\ A\_27b)\ (ap\ (c\_2Eoption\_2ESOME \\ & A\_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow (\forall V0x \in A\_27b.(\forall V1y \in A\_27c.(\forall V2f \in \\ & ((A\_27a^{A\_27c})^{A\_27b}).((ap\ (ap\ (c\_2Epair\_2Epair\_CASE\ A\_27a\ A\_27b \\ & A\_27c)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27b\ A\_27c)\ V0x)\ V1y))\ V2f) = (ap \\ & (ap\ V2f\ V0x)\ V1y)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0x \in A\_27a. (\forall V1y \in A\_27a. (((ap\ (c\_2Epath\_2Estopped\_at \\ & \quad A\_27a\ A\_27b)\ V0x) = (ap\ (c\_2Epath\_2Estopped\_at\ A\_27a\ A\_27b)\ V1y)) \Leftrightarrow \\ & \quad (V0x = V1y)))) \end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0x \in A\_27a. (\forall V1r \in A\_27b. (\forall V2p \in (ty\_2Epath\_2Epath \\ & \quad A\_27a\ A\_27b). (\forall V3y \in A\_27a. (\forall V4s \in A\_27b. (\forall V5q \in \\ & \quad (ty\_2Epath\_2Epath\ A\_27a\ A\_27b). (((ap\ (ap\ (ap\ (c\_2Epath\_2Epcns \\ & \quad A\_27a\ A\_27b)\ V0x)\ V1r)\ V2p) = (ap\ (ap\ (ap\ (c\_2Epath\_2Epcns\ A\_27a \\ & \quad A\_27b)\ V3y)\ V4s)\ V5q)) \Leftrightarrow ((V0x = V3y) \wedge ((V1r = V4s) \wedge (V2p = V5q)))))))))) \end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & \quad nonempty\ A\_27c \Rightarrow (\forall V0proj \in (A\_27b^{A\_27a}). (\forall V1f \in ( \\ & \quad (ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c))^{A\_27a}). \\ & \quad (\forall V2s \in A\_27a. ((ap\ (ap\ (ap\ (c\_2Epath\_2Eunfold\ A\_27b\ A\_27c \\ & \quad A\_27a)\ V0proj)\ V1f)\ V2s) = (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_CASE \\ & \quad (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c)\ (ty\_2Epath\_2Epath\ A\_27b\ A\_27c)) \\ & \quad (ap\ V1f\ V2s))\ (ap\ (c\_2Epath\_2Estopped\_at\ A\_27b\ A\_27c)\ (ap\ V0proj \\ & \quad V2s)))\ (\lambda V3v \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c). (ap\ (ap\ (c\_2Epair\_2Epair\_CASE \\ & \quad (ty\_2Epath\_2Epath\ A\_27b\ A\_27c)\ A\_27a\ A\_27c)\ V3v)\ (\lambda V4s.27 \in \\ & \quad A\_27a. (\lambda V5l \in A\_27c. (ap\ (ap\ (ap\ (c\_2Epath\_2Epcns\ A\_27b\ A\_27c) \\ & \quad (ap\ V0proj\ V2s))\ V5l)\ (ap\ (ap\ (ap\ (c\_2Epath\_2Eunfold\ A\_27b\ A\_27c \\ & \quad A\_27a)\ V0proj)\ V1f)\ V4s.27)))))))))) \end{aligned} \tag{35}$$

### Theorem 1

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & \quad nonempty\ A\_27c \Rightarrow (\forall V0proj \in (A\_27b^{A\_27a}). (\forall V1f \in ( \\ & \quad (ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c))^{A\_27a}). \\ & \quad (\forall V2x \in A\_27a. (\forall V3v1 \in A\_27a. (\forall V4v2 \in A\_27c. \\ & \quad (((ap\ V1f\ V2x) = (c\_2Eoption\_2ENONE\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c))) \Rightarrow \\ & \quad ((ap\ (ap\ (ap\ (c\_2Epath\_2Eunfold\ A\_27b\ A\_27c\ A\_27a)\ V0proj)\ V1f) \\ & \quad V2x) = (ap\ (c\_2Epath\_2Estopped\_at\ A\_27b\ A\_27c)\ (ap\ V0proj\ V2x)))) \wedge \\ & \quad (((ap\ V1f\ V2x) = (ap\ (c\_2Eoption\_2ESOME\ (ty\_2Epair\_2Eprod\ A\_27a \\ & \quad A\_27c))\ (ap\ (ap\ (c\_2Epair\_2E.2C\ A\_27a\ A\_27c)\ V3v1)\ V4v2))) \Rightarrow ((ap \\ & \quad (ap\ (ap\ (c\_2Epath\_2Eunfold\ A\_27b\ A\_27c\ A\_27a)\ V0proj)\ V1f)\ V2x) = \\ & \quad (ap\ (ap\ (ap\ (c\_2Epath\_2Epcns\ A\_27b\ A\_27c)\ (ap\ V0proj\ V2x))\ V4v2) \\ & \quad (ap\ (ap\ (ap\ (c\_2Epath\_2Eunfold\ A\_27b\ A\_27c\ A\_27a)\ V0proj)\ V1f)\ V3v1)))))))))) \end{aligned}$$