

thm_2Epatricia_2EADD_LIST_TO_EMPTY_IS_PTREE (TMU8Sh2pYj67Gz8shxqNbkYMBzV57BjGKAj)

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Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Epatricia_2Eptree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Epatricia_2Eptree A0) \quad (1)$$

Let $c_2Epatricia_2EEmpty : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Epatricia_2EEmpty A_27a \in (ty_2Epatricia_2Eptree A_27a) \quad (2)$$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty ty_2Eenum_2Eenum \quad (3)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (4)$$

Let $c_2Epatricia_2EADD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Epatricia_2EADD A_27a \in (((ty_2Epatricia_2Eptree A_27a)^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum A_27a)})^{(ty_2Epatricia_2Eptree A_27a)}) \quad (5)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (6)$$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EFOLDL A_27a A_27b \in (((A_27b^{(ty_2Elist_2Elist A_27a)})^{A_27b})^{(A_27b^{A_27a})^{A_27b}}) \quad (7)$$

Definition 3 We define $c_Epatricia_EADD_LIST$ to be $\lambda A_27a : \iota.(ap (c_Elist_EFOLDL (ty_Epair_Eprod ty_Eenum_Eenum A_27a) (c_Epatricia_EIS_PTREE A_27a) V0t)) \Rightarrow (p (ap (c_Epatricia_EADD_LIST A_27a) V0t) V1l))$.
Let $c_Epatricia_EIS_PTREE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Epatricia_EIS_PTREE A_27a \in (2^{(ty_Epatricia_Eptree A_27a)}) \quad (8)$$

Definition 4 We define c_Ebool_E21 to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_Emin_E3D (2^{A_27a}) V0t) P) V1t))$.

Definition 5 We define c_Ebool_E2F to be $(ap (c_Ebool_E21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_Emin_E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 7 We define $c_Ebool_E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E21 2) (\lambda V2t \in 2.V2t)))$

Definition 8 We define $c_Ebool_E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E21 2) (\lambda V2t \in 2.V2t)))$

Definition 9 We define c_Ebool_E7E to be $(\lambda V0t \in 2.(ap (ap c_Emin_E3D_3D_3E V0t) c_Ebool_E2F V0t))$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg (\neg (p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\ & p V0t)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (p (ap (c_Epatricia_EIS_PTREE A_27a) (c_Epatricia_EEmpty A_27a))) \quad (13)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in (ty_Epatricia_Eptree A_27a).(\forall V1l \in (ty_Elist_Elist (ty_Epair_Eprod ty_Eenum_Eenum A_27a) (c_Epatricia_EIS_PTREE A_27a) V0t)) \Rightarrow (p (ap \\ & (c_Epatricia_EIS_PTREE A_27a) (ap (ap (c_Epatricia_EADD_LIST A_27a) V0t) V1l)))))) \end{aligned} \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (15)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (16)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (17)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (18)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (19)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (20)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (21)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (22)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (23)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (24)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (25)$$

Theorem 1

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0l \in (\text{ty_2Elist_2Elist} \\ (\text{ty_2Epair_2Eprod } \text{ty_2Enum_2Enum } A_{27a})). (\text{p } (\text{ap } (\text{c_2Epatricia_2EIS_PTREE} \\ A_{27a}) (\text{ap } (\text{ap } (\text{c_2Epatricia_2EADD_LIST } A_{27a}) (\text{c_2Epatricia_2EEmpty} \\ A_{27a})) V0l))))))$$