

thm_2Epatricia_2ECARD_LIST_TO_SET (TMW66nQxM4UjwdhfU9WF9yU5QNvMELG2F2R)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{2}$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \tag{3}$$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_2F$

Let $c_2Elist_2EALL_DISTINCT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EALL_DISTINCT\ A_27a \in (2^{(ty_2Elist_2Elist\ A_27a)}) \tag{4}$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \tag{5}$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (6)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)}) \quad (7)$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2)))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (9)$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 9 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 10 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 11 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 12 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(\lambda V3t3 \in 2))))$

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (10)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (11)$$

Definition 14 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2z \in 2)))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (12)$$

Definition 15 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Ebool_2E_21 (2^{A_27a})) (c_2Epred_set_2E_21 (2^{A_27a})))$.
Let $c_2Epred_set_2E_21 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Epred_set_2E_21\ A_27a \in (ty_2Enum_2Enum^{(2^{A_27a})}) \quad (13)$$

Definition 16 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_21 (2^{A_27a})) (c_2Epred_set_2E_21 (2^{A_27a})))$.
Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (14)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (15)$$

Definition 17 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num (c_2Enum_2ESUC_REP m))$.
Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (22)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (23)$$

Assume the following.

$$2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow 2.(((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))) \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_{.27a}.(\forall V3x_{.27} \in A_{.27a}.(\forall V4y \in A_{.27a}. \\ & (\forall V5y_{.27} \in A_{.27a}.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))) \Rightarrow ((ap (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) V1Q) V3x_{.27}) \\ & V5y_{.27})))))))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow ((\forall V0t1 \in A_{.27a}.(\forall V1t2 \in \\ & A_{.27a}.((ap (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) c_{.2Ebool_{.2ET}}) V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{.27a}.(\forall V3t2 \in A_{.27a}.((ap \\ & (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) c_{.2Ebool_{.2EF}}) V2t1) V3t2) = V3t2)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (((ap (c_{.2Elist_{.2ELENGTH}} A_{.27a}) \\ & (c_{.2Elist_{.2ENIL}} A_{.27a})) = c_{.2Enum_{.2E0}}) \wedge (\forall V0h \in A_{.27a}.(\\ & \forall V1t \in (ty_{.2Elist_{.2Elist}} A_{.27a}).((ap (c_{.2Elist_{.2ELENGTH}} \\ & A_{.27a}) (ap (ap (c_{.2Elist_{.2ECONS}} A_{.27a}) V0h) V1t)) = (ap c_{.2Enum_{.2ESUC}} \\ & (ap (c_{.2Elist_{.2ELENGTH}} A_{.27a}) V1t)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{(ty_{.2Elist_{.2Elist}} A_{.27a})}). \\ & (((p (ap V0P (c_{.2Elist_{.2ENIL}} A_{.27a}))) \wedge (\forall V1t \in (ty_{.2Elist_{.2Elist}} \\ & A_{.27a}).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A_{.27a}.(p (ap V0P (ap (ap (\\ & c_{.2Elist_{.2ECONS}} A_{.27a}) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty_{.2Elist_{.2Elist}} \\ & A_{.27a}).(p (ap V0P V3l)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (((p\ (ap\ (c.2Elist.2EALL_DISTINCT \\ & A.27a)\ (c.2Elist.2ENIL\ A.27a))) \Leftrightarrow True) \wedge (\forall V0h \in A.27a. \\ \forall V1t \in (ty.2Elist.2Elist\ A.27a). & ((p\ (ap\ (c.2Elist.2EALL_DISTINCT \\ & A.27a)\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27a)\ V0h)\ V1t))) \Leftrightarrow ((\neg(p\ (ap\ (ap \\ & (c.2Ebool.2EIN\ A.27a)\ V0h)\ (ap\ (c.2Elist.2ELIST_TO_SET\ A.27a) \\ & V1t)))) \wedge (p\ (ap\ (c.2Elist.2EALL_DISTINCT\ A.27a)\ V1t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0l \in (ty.2Elist.2Elist \\ & A.27a). (p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ (ap\ (c.2Elist.2ELIST_TO_SET \\ & A.27a)\ V0l)))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0h \in A.27b. (\forall V1t \in (ty.2Elist.2Elist\ A.27b). ((\\ & (ap\ (c.2Elist.2ELIST_TO_SET\ A.27a)\ (c.2Elist.2ENIL\ A.27a)) = \\ & (c.2Epred_set.2EEMPTY\ A.27a)) \wedge ((ap\ (c.2Elist.2ELIST_TO_SET \\ & A.27b)\ (ap\ (ap\ (c.2Elist.2ECONS\ A.27b)\ V0h)\ V1t)) = (ap\ (ap\ (c.2Epred_set.2EINSERT \\ & A.27b)\ V0h)\ (ap\ (c.2Elist.2ELIST_TO_SET\ A.27b)\ V1t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & ((ap\ (c.2Epred_set.2ECARD\ A.27a) \\ & (c.2Epred_set.2EEMPTY\ A.27a)) = c.2Enum.2E0) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0s \in (2^{A.27a}). ((p\ (ap \\ & (c.2Epred_set.2EFINITE\ A.27a)\ V0s)) \Rightarrow (\forall V1x \in A.27a. ((\\ & ap\ (c.2Epred_set.2ECARD\ A.27a)\ (ap\ (ap\ (c.2Epred_set.2EINSERT \\ & A.27a)\ V1x)\ V0s)) = (ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ ty.2Enum.2Enum) \\ & (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1x)\ V0s))\ (ap\ (c.2Epred_set.2ECARD \\ & A.27a)\ V0s))\ (ap\ c.2Enum.2ESUC\ (ap\ (c.2Epred_set.2ECARD\ A.27a) \\ & V0s)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty.2Enum.2Enum. (\forall V1n \in ty.2Enum.2Enum. (\\ & ((ap\ c.2Enum.2ESUC\ V0m) = (ap\ c.2Enum.2ESUC\ V1n)) \Leftrightarrow (V0m = V1n)))) \end{aligned} \quad (34)$$

Theorem 1

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0l \in (ty.2Elist.2Elist \\ & A.27a). ((p\ (ap\ (c.2Elist.2EALL_DISTINCT\ A.27a)\ V0l)) \Rightarrow ((ap\ (\\ & c.2Epred_set.2ECARD\ A.27a)\ (ap\ (c.2Elist.2ELIST_TO_SET\ A.27a) \\ & V0l)) = (ap\ (c.2Elist.2ELENGTH\ A.27a)\ V0l)))) \end{aligned}$$