

thm_2Epatricia_2ECARD_NUMSET_OF_PTREE (TMQJxTPH5qmPLVjYta2B5wKfQmb5ZxmJtF)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{2}$$

Let $ty_2Epatricia_2Eptree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Epatricia_2Eptree\ A0) \tag{3}$$

Let $c_2Epatricia_2ETRAVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Epatricia_2ETRAVERSE\ A_27a \in ((ty_2Elist_2Elist\ ty_2Enum_2Enum)^{(ty_2Epatricia_2Eptree\ A_27a)}) \tag{4}$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \tag{5}$$

Definition 8 We define $c_2Epatricia_2ESIZE$ to be $\lambda A_27a : \iota. \lambda V0t \in (ty_2Epatricia_2Eptree A_27a). (ap$
 Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (6)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)}) \quad (7)$$

Definition 9 We define $c_2Epatricia_2ENUMSET_OF_PTREE$ to be $\lambda V0t \in (ty_2Epatricia_2Eptree\ ty_2E$
 Let $c_2Epatricia_2EIS_PTREE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Epatricia_2EIS_PTREE\ A_27a \in (2^{(ty_2Epatricia_2Eptree\ A_27a)}) \quad (8)$$

Let $c_2Epred_set_2ECARD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Epred_set_2ECARD\ A_27a \in (ty_2Enum_2Enum^{(2^{A_27a})}) \quad (9)$$

Let $c_2Elist_2EALL_DISTINCT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EALL_DISTINCT\ A_27a \in (2^{(ty_2Elist_2Elist\ A_27a)}) \quad (10)$$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\ & p\ V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (16)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (17)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0t \in (ty_2Epatricia_2Eptree A_{27a}).((p (ap (c_2Epatricia_2EIS_PTREE A_{27a}) V0t)) \Rightarrow (p (ap (c_2Elist_2EALL_DISTINCT ty_2Enum_2Enum) (ap (c_2Epatricia_2ETRAVERSE A_{27a}) V0t)))))) \quad (18)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0l \in (ty_2Elist_2Elist A_{27a}).((p (ap (c_2Elist_2EALL_DISTINCT A_{27a}) V0l)) \Rightarrow ((ap (c_2Epred_set_2ECARD A_{27a}) (ap (c_2Elist_2ELIST_TO_SET A_{27a}) V0l)) = (ap (c_2Elist_2ELENGTH A_{27a}) V0l)))))) \quad (19)$$

Theorem 1

$$(\forall V0t \in (ty_2Epatricia_2Eptree ty_2Eone_2Eone).((p (ap (c_2Epatricia_2EIS_PTREE ty_2Eone_2Eone) V0t)) \Rightarrow ((ap (c_2Epred_set_2ECARD ty_2Enum_2Enum) (ap c_2Epatricia_2ENUMSET_OF_PTREE V0t)) = (ap (c_2Epatricia_2ESIZE ty_2Eone_2Eone) V0t))))))$$