thm_2Epatricia_2EEMPTY__IS__PTREE (TMFLkMzpBh5HDxMDpBwFupXV4h2u5hxPreS)

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Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 7 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 9 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{8}$$

Definition 10 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{9}$$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{10}$$

Definition 11 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 12 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Definition 13 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Definition 14 *We define* c_2Ebit_2EMOD__2EXP__EQ *to be* $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1a \in ty\_2Enum$

Let $ty\_2Epatricia\_2Eptree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Epatricia\_2Eptree\ A0) \tag{11}$$

Let $c\_2Epatricia\_2EEVERY\_\_LEAF : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EEVERY\_\_LEAF\ A\_27a \in ((2^{(ty\_2Epatricia\_2Eptree\ A\_27a)})^{((2^{A\_27a})^{ty\_2Enum\_2Enum})}) \tag{12}$$

Definition 15 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t)).$

Definition 16 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 17 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 18 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 19 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 20 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 21 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Let $c\_2Epatricia\_2EBranch : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EBranch\ A\_27a \in (((((ty\_2Epatricia\_2Eptree\ A\_27a)^{(ty\_2Epatricia\_2Eptree\ A\_27a)})^{(ty\_2Epatricia\_2Eptree\ A\_27a)})^{ty\_2Enum\_2Enum}) \tag{13}$$

Let $c\_2Epatricia\_2ELeaf : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2ELeaf\ A\_27a \in (((ty\_2Epatricia\_2Eptree\ A\_27a)^{A\_27a})^{ty\_2Enum\_2Enum}) \tag{14}$$

Let $c\_2Epatricia\_2EEmpty : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EEmpty\ A\_27a \in (ty\_2Epatricia\_2Eptree\ A\_27a) \tag{15}$$

Let $c\_2Epatricia\_2EIS\_\_PTREE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EIS\_\_PTREE\ A\_27a \in (2^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \tag{16}$$

Assume the following.

$$True \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ A\_27a)\ (c\_2Epatricia\_2EEmpty\ A\_27a))) \Leftrightarrow True) \wedge ((\forall V0k \in ty\_2Enum\_2Enum.(\forall V1d \in A\_27a.((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ A\_27a)\ (ap\ (ap\ (c\_2Epatricia\_2ELeaf\ A\_27a)\ V0k)\ V1d))) \Leftrightarrow True))) \wedge (\forall V2p \in ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.(\forall V4l \in (ty\_2Epatricia\_2Eptree\ A\_27a).(\forall V5r \in (ty\_2Epatricia\_2Eptree\ A\_27a).((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Epatricia\_2EBranch\ A\_27a)\ V2p)\ V3m)\ V4l)\ V5r))) \Leftrightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V2p)\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V3m))) \wedge ((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ A\_27a)\ V4l)) \wedge ((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ A\_27a)\ V5r)) \wedge ((\neg (V4l = (c\_2Epatricia\_2EEmpty\ A\_27a))) \wedge ((\neg (V5r = (c\_2Epatricia\_2EEmpty\ A\_27a))) \wedge ((p\ (ap\ (ap\ (c\_2Epatricia\_2EEVERY\_\_LEAF\ A\_27a)\ (\lambda V6k \in ty\_2Enum\_2Enum.(\lambda V7d \in A\_27a.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (ap\ \mathsf{c\_2Ebit\_2EMOD\_\_2EXP\_\_EQ}\ V3m)\ V6k)\ V2p))\ (ap\ (ap\ \mathsf{c\_2Ebit\_2EBIT}\ V3m)\ V6k)))))\ V4l)) \wedge (p\ (ap\ (ap\ (c\_2Epatricia\_2EEVERY\_\_LEAF\ A\_27a)\ (\lambda V8k \in ty\_2Enum\_2Enum.(\lambda V9d \in A\_27a.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (ap\ \mathsf{c\_2Ebit\_2EMOD\_\_2EXP\_\_EQ}\ V3m)\ V8k)\ V2p))\ (ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ \mathsf{c\_2Ebit\_2EBIT}\ V3m)\ V8k))))))\ V5r)))))))))))))) \tag{18}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ A\_27a)\ (c\_2Epatricia\_2EEmpty\ A\_27a)))$$