

thm\_2Epatricia\_2EIN\_\_NUMSET\_\_OF\_\_PTREE  
(TMc9xF9K1WkeD9sBwqSxD6pABQuHeR8sQnm)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{2}$$

Let  $ty\_2Eenum\_2Eenum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eenum\_2Eenum \tag{3}$$

Let  $ty\_2Epatricia\_2Eptree : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Epatricia\_2Eptree\ A0) \tag{4}$$

Let  $c\_2Epatricia\_2EPEEK : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EPEEK\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Eenum\_2Eenum})^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \tag{5}$$

Let  $c\_2Eoption\_2EIS\_SOME : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_SOME\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{6}$$

**Definition 8** We define  $c\_2Epatricia\_2EIN\_PTREE$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1t \in (ty\_2Epatricia\_2Etree A\_27a)$ .  
Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (7)$$

Let  $c\_2Epatricia\_2ETRAVERSE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2ETRAVERSE\ A\_27a \in ((ty\_2Elist\_2Elist\ ty\_2Enum\_2Enum)^{(ty\_2Epatricia\_2Etree\ A\_27a)}) \quad (8)$$

Let  $c\_2Elist\_2ELIST\_TO\_SET : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELIST\_TO\_SET\ A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (9)$$

**Definition 9** We define  $c\_2Epatricia\_2ENUMSET\_OF\_PTREE$  to be  $\lambda V0t \in (ty\_2Epatricia\_2Etree\ ty\_2Enum\_2Enum\ A\_27a)$ .

**Definition 10** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(\lambda p\ V1f\ V0x)))$ .

Let  $c\_2Epatricia\_2EIS\_PTREE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EIS\_PTREE\ A\_27a \in (2^{(ty\_2Epatricia\_2Etree\ A\_27a)}) \quad (10)$$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg ( \\ & p\ V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (16)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27)))))))) \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0t \in (ty\_2Epatricia\_2Eptree \\ & A\_27a). (\forall V1k \in ty\_2Enum\_2Enum. ((p (ap (c\_2Epatricia\_2EIS\_PTREE \\ & A\_27a) V0t)) \Rightarrow ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Enum\_2Enum) V1k) ( \\ & ap (c\_2Elist\_2ELIST\_TO\_SET ty\_2Enum\_2Enum) (ap (c\_2Epatricia\_2ETRAVERSE \\ & A\_27a) V0t)))) \Leftrightarrow (p (ap (c\_2Eoption\_2EIS\_SOME A\_27a) (ap (ap (c\_2Epatricia\_2EPEEK \\ & A\_27a) V0t) V1k)))))))) \quad (18) \end{aligned}$$

**Theorem 1**

$$\begin{aligned} & (\forall V0t \in (ty\_2Epatricia\_2Eptree ty\_2Eone\_2Eone). (\forall V1n \in \\ & ty\_2Enum\_2Enum. ((p (ap (c\_2Epatricia\_2EIS\_PTREE ty\_2Eone\_2Eone) \\ & V0t)) \Rightarrow ((p (ap (ap (c\_2Ebool\_2EIN ty\_2Enum\_2Enum) V1n) (ap c\_2Epatricia\_2ENUMSET\_OF\_PTREE \\ & V0t)))) \Leftrightarrow (p (ap (ap c\_2Epatricia\_2EIN\_PTREE V1n) V0t)))))) \end{aligned}$$