

thm_2Epatricia_2EIN__PTREE__REMOVE
(TMYAqS877a5ER8G4WLGpX4NSoBQWj71A99e)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 6 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A-27b})^{A-27a})^2}) \tag{3}$$

Definition 9 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS_sum$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (5)$$

Definition 10 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Let $c_2Eoption_2EIS_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2EIS_SOME\ A_27a \in (2^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (6)$$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \quad (7)$$

Let $ty_2Epatricia_2Eptree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Epatricia_2Eptree\ A0) \quad (8)$$

Let $c_2Epatricia_2EPEEK : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Epatricia_2EPEEK\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{ty_2Eenum_2Eenum})^{(ty_2Epatricia_2Eptree\ A_27a)}) \quad (9)$$

Definition 11 We define $c_2Epatricia_2EIN_PTREE$ to be $\lambda V0n \in ty_2Eenum_2Eenum. \lambda V1t \in (ty_2Epatricia_2Eptree\ n)$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P\ x) \text{ then } (the\ (\lambda x. x \in A) \setminus P)$ of type $\iota \Rightarrow \iota$.

Definition 13 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone. V0x))$

Definition 14 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS\ A_27a)\ V0e)$

Definition 15 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (c_2Eone_2Eone))$

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (V0t1 \wedge V2t2))))$

Let $c_2Epatricia_2EREMOVE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Epatricia_2EREMOVE\ A_27a \in ((ty_2Epatricia_2Eptree\ A_27a)^{ty_2Eenum_2Eenum})^{(ty_2Epatricia_2Eptree\ A_27a)} \quad (10)$$

Let $c_2Epatricia_2EIS_PTREE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Epatricia_2EIS_PTREE\ A_27a \in (2^{(ty_2Epatricia_2Eptree\ A_27a)}) \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (17)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (21)$$

Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow \\ (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow ((\forall V0x \in A_27a. ((p\ (ap\ (c_2Eoption_2EIS_SOME \\ A_27a)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x))) \Leftrightarrow True)) \wedge ((p\ (ap\ (c_2Eoption_2EIS_SOME \\ A_27a)\ (c_2Eoption_2ENONE\ A_27a))) \Leftrightarrow False)) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0t \in (ty_2Epatricia_2Eptree \\ A_27a). (\forall V1k \in ty_2Enum_2Enum. (\forall V2j \in ty_2Enum_2Enum. \\ ((p\ (ap\ (c_2Epatricia_2EIS_PTREE\ A_27a)\ V0t)) \Rightarrow ((ap\ (ap\ (c_2Epatricia_2EPEEK \\ A_27a)\ (ap\ (ap\ (c_2Epatricia_2EREMOVE\ A_27a)\ V0t)\ V1k))\ V2j) = (\\ ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a))\ (ap \\ (ap\ (c_2Emin_2E_3D\ ty_2Enum_2Enum)\ V1k)\ V2j))\ (c_2Eoption_2ENONE \\ A_27a))\ (ap\ (ap\ (c_2Epatricia_2EPEEK\ A_27a)\ V0t)\ V2j)))))) \end{aligned} \quad (26)$$

Theorem 1

$$\begin{aligned} (\forall V0t \in (ty_2Epatricia_2Eptree\ ty_2Eone_2Eone). (\forall V1m \in \\ ty_2Enum_2Enum. (\forall V2n \in ty_2Enum_2Enum. ((p\ (ap\ (c_2Epatricia_2EIS_PTREE \\ ty_2Eone_2Eone)\ V0t)) \Rightarrow ((p\ (ap\ (ap\ c_2Epatricia_2EIN_PTREE\ V2n) \\ (ap\ (ap\ (c_2Epatricia_2EREMOVE\ ty_2Eone_2Eone)\ V0t)\ V1m))) \Leftrightarrow (\\ (\neg(V2n = V1m)) \wedge (p\ (ap\ (ap\ c_2Epatricia_2EIN_PTREE\ V2n)\ V0t)))))) \end{aligned}$$