thm_2Epatricia_2EIS__EMPTY__def (TMRk4NuJjhgmy4meNfhgNDvYmq137KQKnoa)

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Let $ty\_2Epatricia\_2Eptree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Epatricia\_2Eptree\ A0) \quad (1)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let $c\_2Epatricia\_2EBranch : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EBranch\ A\_27a \in (((((ty\_2Epatricia\_2Eptree\ A\_27a)^{(ty\_2Epatricia\_2Eptree\ A\_27a)})^{(ty\_2Epatricia\_2Eptree\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (3)$$

Let $c\_2Epatricia\_2ELeaf : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2ELeaf\ A\_27a \in (((ty\_2Epatricia\_2Eptree\ A\_27a)^{A\_27a})^{ty\_2Enum\_2Enum}) \quad (4)$$

Let $c\_2Epatricia\_2EEmpty : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EEmpty\ A\_27a \in (ty\_2Epatricia\_2Eptree\ A\_27a) \quad (5)$$

Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 8 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

Definition 9 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A\_$

Let $c\_2Epatricia\_2Eptree\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epat$$
$$A\_27a\ A\_27b \in ((((A\_27b^{((((A\_27b^{(ty\_2Epatricia\_2Eptree\ A\_27a)})^{(ty\_2Epatricia\_2Eptree\ A\_27a)})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2} \quad (6)$$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 12 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 13 *We define* c_2Erelation_2EWF *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}$

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \quad (7)$$

Definition 14 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 15 *We define* c_2Erelation_2ERESTRICT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1$

Definition 16 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 17 *We define* c_2Erelation_2Eapprox *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 18 *We define* c_2Erelation_2Ethe__fun *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 19 *We define* c_2Erelation_2EWFREC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 20 *We define* c_2Epatricia_2EIS__EMPTY *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Erelation\_2EWFREC}\ (ty\_2$

Definition 21 *We define* c_2Erelation_2EEMPTY__REL *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27a.\mathsf{c\_2E}$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0v \in A\_27b.(\forall V1f \in ((A\_27b^{A\_27a})^{ty\_2Enum\_2Enum}). (\forall V2f1 \in ((((A\_27b^{(ty\_2Epatricia\_2Eptree\ A\_27a)})^{(ty\_2Epatricia\_2Eptree\ A\_27a)})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enu}). ((ap\ (ap\ (ap\ (ap\ (c\_2Epatricia\_2Eptree\_\_CASE\ A\_27a\ A\_27b)\ (c\_2Epatricia\_2EEmpty\ A\_27a))\ V0v)\ V1f)\ V2f1) = V0v)))) \wedge ((\forall V3a0 \in ty\_2Enum\_2Enum. (\forall V4a1 \in A\_27a.(\forall V5v \in A\_27b.(\forall V6f \in ((A\_27b^{A\_27a})^{ty\_2Enum\_2Enum}). (\forall V7f1 \in ((((A\_27b^{(ty\_2Epatricia\_2Eptree\ A\_27a)})^{(ty\_2Epatricia\_2Eptree\ A\_27a)})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enu}). ((ap\ (ap\ (ap\ (ap\ (c\_2Epatricia\_2Eptree\_\_CASE\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Epatricia\_2ELeaf\ A\_27a)\ V3a0)\ V4a1))\ V5v)\ V6f)\ V7f1) = (ap\ (ap\ V6f\ V3a0)\ V4a1))))))) \wedge (\forall V8a0 \in ty\_2Enum\_2Enum.(\forall V9a1 \in ty\_2Enum\_2Enum.(\forall V10a2 \in (ty\_2Epatricia\_2Eptree\ A\_27a). (\forall V11a3 \in (ty\_2Epatricia\_2Eptree\ A\_27a).(\forall V12v \in A\_27b.(\forall V13f \in ((A\_27b^{A\_27a})^{ty\_2Enum\_2Enum}).(\forall V14f1 \in ((((A\_27b^{(ty\_2Epatricia\_2Eptree\ A\_27a)})^{(ty\_2Epatricia\_2Eptree\ A\_27a)})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}). ((ap\ (ap\ (ap\ (ap\ (c\_2Epatricia\_2Eptree\_\_CASE\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (ap\ (c\_2Epatricia\_2EBranch\ A\_27a)\ V8a0)\ V9a1)\ V10a2)\ V11a3)) V12v)\ V13f)\ V14f1) = (ap\ (ap\ (ap\ (ap\ V14f1\ V8a0)\ V9a1)\ V10a2)\ V11a3)))))))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ A\_27a)\ (\mathsf{c\_2Erelation\_2EEMPTY\_\_REL}\ A\_27a))) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0M \in ((A\_27b^{A\_27a})^{(A\_27b^{A\_27a})}).(\forall V1R \in ((2^{A\_27a})^{A\_27a}). (\forall V2f \in (A\_27b^{A\_27a}).((V2f = (ap\ (ap\ (\mathsf{c\_2Erelation\_2EWFREC}\ A\_27a\ A\_27b)\ V1R)\ V0M)) \Rightarrow ((p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ A\_27a)\ V1R)) \Rightarrow (\forall V3x \in A\_27a.((ap\ V2f\ V3x) = (ap\ (ap\ V0M\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ERESTRICT}\ A\_27a\ A\_27b)\ V2f)\ V1R)\ V3x))\ V3x)))))))) \tag{11}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0v \in ty\_2Enum\_2Enum.(\forall V1v1 \in A\_27a.(\forall V2v2 \in ty\_2Enum\_2Enum.(\forall V3v3 \in ty\_2Enum\_2Enum.(\forall V4v4 \in (ty\_2Epatricia\_2Eptree\ A\_27a). (\forall V5v5 \in (ty\_2Epatricia\_2Eptree\ A\_27a).(((p\ (ap\ (\mathsf{c\_2Epatricia\_2EIS\_\_EMPTY}\ A\_27a)\ (c\_2Epatricia\_2EEmpty\ A\_27a))) \Leftrightarrow True) \wedge (((p\ (ap\ (\mathsf{c\_2Epatricia\_2EIS\_\_EMPTY}\ A\_27a)\ (ap\ (ap\ (c\_2Epatricia\_2ELeaf\ A\_27a)\ V0v)\ V1v1))) \Leftrightarrow False) \wedge ((p\ (ap\ (\mathsf{c\_2Epatricia\_2EIS\_\_EMPTY}\ A\_27a)\ (ap\ (ap\ (ap\ (ap\ (c\_2Epatricia\_2EBranch\ A\_27a)\ V2v2)\ V3v3)\ V4v4)\ V5v5))) \Leftrightarrow False)))))))))$$