

thm_2Epatricia_2EMEM__TRAVERSE
(TMU8tmf79sV7X7vcS8314vhrzb5kryMTKKj)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (2)$$

Let $ty_2Epatricia_2Eptree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Epatricia_2Eptree A0) \quad (3)$$

Let $c_2Epatricia_2EPEEK : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Epatricia_2EPEEK A_27a \in (((ty_2Eoption_2Eoption A_27a)^{ty_2Enum_2Enum})(ty_2Epatricia_2Eptree A_27a)) \quad (4)$$

Let $c_2Eoption_2EIS_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2EIS_SOME A_27a \in (2^{(ty_2Eoption_2Eoption A_27a)}) \quad (5)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (6)$$

Let $c_2Epatricia_2ETRAVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Epatricia_2ETRAVERSE\ A_27a \in ((ty_2Elist_2Elist\ ty_2Enum_2Enum)^{(ty_2Epatricia_2Eptree\ A_27a)}) \quad (7)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)}) \quad (8)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (9)$$

Definition 8 We define $c_2Epatricia_2EIN_PTREE$ to be $\lambda V0n \in ty_2Enum_2Enum.\lambda V1t \in (ty_2Epatricia_2Eptree\ ty_2Enum_2Enum)$

Definition 9 We define $c_2Epatricia_2ENUMSET_OF_PTREE$ to be $\lambda V0t \in (ty_2Epatricia_2Eptree\ ty_2Enum_2Enum)$

Definition 10 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(\lambda p\ V1f\ V0x)))$

Let $c_2Epatricia_2EIS_PTREE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Epatricia_2EIS_PTREE\ A_27a \in (2^{(ty_2Epatricia_2Eptree\ A_27a)}) \quad (10)$$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\ & p\ V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (16)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in (ty_2Epatricia_2Eptree \\ & A_27a).(\forall V1k \in ty_2Enum_2Enum.((p (ap (c_2Epatricia_2EIS_PTREE \\ & A_27a) V0t)) \Rightarrow ((p (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) V1k) (\\ & ap (c_2Elist_2ELIST_TO_SET ty_2Enum_2Enum) (ap (c_2Epatricia_2ETRAVERSE \\ & A_27a) V0t)))) \Leftrightarrow (p (ap (c_2Eoption_2EIS_SOME A_27a) (ap (ap (c_2Epatricia_2EPEEK \\ & A_27a) V0t) V1k))))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in (ty_2Epatricia_2Eptree ty_2Eone_2Eone).(\forall V1n \in \\ & ty_2Enum_2Enum.((p (ap (c_2Epatricia_2EIS_PTREE ty_2Eone_2Eone) \\ & V0t)) \Rightarrow ((p (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) V1n) (ap c_2Epatricia_2ENUMSET_OF_PTREE \\ & V0t))) \Leftrightarrow (p (ap (ap c_2Epatricia_2EIN_PTREE V1n) V0t)))))) \end{aligned} \quad (19)$$

Theorem 1

$$\begin{aligned} & (\forall V0t \in (ty_2Epatricia_2Eptree ty_2Eone_2Eone).(\forall V1k \in \\ & ty_2Enum_2Enum.((p (ap (c_2Epatricia_2EIS_PTREE ty_2Eone_2Eone) \\ & V0t)) \Rightarrow ((p (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) V1k) (ap (c_2Elist_2ELIST_TO_SET \\ & ty_2Enum_2Enum) (ap (c_2Epatricia_2ETRAVERSE ty_2Eone_2Eone) \\ & V0t)))) \Leftrightarrow (p (ap (ap (c_2Ebool_2EIN ty_2Enum_2Enum) V1k) (ap c_2Epatricia_2ENUMSET_OF_PTREE \\ & V0t)))))) \end{aligned}$$