thm_2Epatricia_2EPEEK__INSERT__PTREE (TMJX8HXGfcPFoBU8Z43wAGaNh5DPs6uKVKw)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \tag{3}$$

Definition 8 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS\_$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (4)$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (5)$$

Definition 9 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_A$

Definition 10 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 11 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (6)$$

Let $ty\_2Epatricia\_2Eptree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Epatricia\_2Eptree\ A0) \quad (7)$$

Let $c\_2Epatricia\_2EPEEK : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EPEEK\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \quad (8)$$

Let $c\_2Epatricia\_2EIS\_\_PTREE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EIS\_\_PTREE\ A\_27a \in (2^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \quad (9)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (10)$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (11)$$

Definition 12 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epatricia\_2EADD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EADD\ A\_27a \in (((ty\_2Epatricia\_2Eptree\ A\_27a)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ A\_27a)})^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \quad (12)$$

Definition 13 *We define* c_2Epatricia_2EINSERT__PTREE *to be* $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1t \in (ty\_2E$

Assume the following.

$$(\forall V0v \in ty\_2Eone\_2Eone.(V0v = \mathsf{c\_2Eone\_2Eone})) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in (ty\_2Epatricia\_2Eptree\ A\_27a).(\forall V1k \in ty\_2Enum\_2Enum.(\forall V2d \in A\_27a.(\forall V3j \in ty\_2Enum\_2Enum.((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ A\_27a)\ V0t)) \Rightarrow ((ap\ (ap\ (c\_2Epatricia\_2EPEEK\ A\_27a)\ (ap\ (ap\ (c\_2Epatricia\_2EADD\ A\_27a)\ V0t)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ A\_27a)\ V1k)\ V2d)))\ V3j) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Enum\_2Enum)\ V1k)\ V3j))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2d))\ (ap\ (ap\ (c\_2Epatricia\_2EPEEK\ A\_27a)\ V0t)\ V3j)))))))) \tag{14}$$

Assume the following.

$$(\forall V0v \in ty\_2Eone\_2Eone.(\forall V1t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V2n \in ty\_2Enum\_2Enum.((ap\ (ap\ (c\_2Epatricia\_2EADD\ ty\_2Eone\_2Eone)\ V1t)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ ty\_2Eone\_2Eone)\ V2n)\ V0v)) = (ap\ (ap\ \mathsf{c\_2Epatricia\_2EINSERT\_\_PTREE}\ V2n)\ V1t))))) \tag{15}$$

Theorem 1

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1k \in ty\_2Enum\_2Enum.(\forall V2j \in ty\_2Enum\_2Enum.((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V0t)) \Rightarrow ((ap\ (ap\ (c\_2Epatricia\_2EPEEK\ ty\_2Eone\_2Eone)\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EINSERT\_\_PTREE}\ V1k)\ V0t))\ V2j) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Eoption\_2Eoption\ ty\_2Eone\_2Eone))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Enum\_2Enum)\ V1k)\ V2j))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ ty\_2Eone\_2Eone)\ \mathsf{c\_2Eone\_2Eone}))\ (ap\ (ap\ (c\_2Epatricia\_2EPEEK\ ty\_2Eone\_2Eone)\ V0t)\ V2j)))))))$$