thm_2Epatricia_2EPERM__ADD (TMbKiww1io8VqnWtt43xV7pcseBDGT5M21b)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{2}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{3}$$

Let $ty\_2Epatricia\_2Eptree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Epatricia\_2Eptree\ A0) \tag{4}$$

Let $c\_2Epatricia\_2ETRANSFORM : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epatricia\_2ETRANSFORM\ A\_27a\ A\_27b \in (((ty\_2Epatricia\_2Eptree\ A\_27a)^{(ty\_2Epatricia\_2Eptree\ A\_27b)})^{(A\_27a^{A\_27b})}) \tag{5}$$

Let $c\_2Elist\_2EALL\_\_DISTINCT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EALL\_\_DISTINCT\ A\_27a \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{6}$$

Let $c\_2Elist\_2ELIST\_\_TO\_\_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{7}$$

Definition 4 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{8}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{9}$$

Definition 8 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{10}$$

Let $c\_2Epatricia\_2EADD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EADD\ A\_27a \in (((ty\_2Epatricia\_2Eptree\ A\_27a)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ A\_27a)})^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \tag{11}$$

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \tag{12}$$

Let $c\_2Epred\_\_set\_2ECHOICE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epred\_\_set\_2ECHOICE\ A\_27a \in (A\_27a^{(2^{A\_27a})}) \tag{13}$$

Definition 9 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 11 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{14}$$

Definition 12 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 13 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 14 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Definition 15 *We define* c_2Epred__set_2EDELETE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap\ (a$

Definition 16 *We define* c_2Epred__set_2EREST *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (ap$ (c_2Epred__set_2E

Definition 17 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$ *of type* $\iota \Rightarrow \iota$.

Definition 18 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 19 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_21 (2

Definition 20 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 21 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap$ (c_2Ecombin_2ES $A\_27a\ (A\_27a^{A\_27a})\ A$

Definition 22 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 23 *We define* c_2Erelation_2EWF *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap$ (c_2Ebool_2E_21

Definition 24 *We define* c_2Erelation_2ERESTRICT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1$

Definition 25 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 26 *We define* c_2Erelation_2Eapprox *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 27 *We define* c_2Erelation_2Ethe__fun *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1$

Definition 28 *We define* c_2Erelation_2EWFREC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1$

Definition 29 *We define* c_2Elist_2ESET__TO__LIST *to be* $\lambda A\_27a : \iota.(ap\ (ap$ (c_2Erelation_2EWFREC $(2^{A\_}$

Definition 30 *We define* c_2Ecombin_2EC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Let $c\_2Elist\_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27b})^{((A\_27b^{A\_27a})^{A\_27b})}) \tag{15}$$

Let $c\_2Epatricia\_2ETRAVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2ETRAVERSE\ A\_27a \in ((ty\_2Elist\_2Elist\ ty\_2Enum\_2Enum)^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \tag{16}$$

Definition 31 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c.$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{17}$$

Definition 32 *We define* c_2Epatricia_2ENUMSET__OF__PTREE *to be* $\lambda V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{18}$$

Let $c\_2Epatricia\_2EPEEK : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EPEEK\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \tag{19}$$

Let $c\_2Eoption\_2EIS\_\_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_\_SOME\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{20}$$

Definition 33 *We define* c_2Epatricia_2EIN__PTREE *to be* $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1t \in (ty\_2Epatri$

Definition 34 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 35 *We define* c_2Epatricia_2EINSERT__PTREE *to be* $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1t \in (ty\_2E$

Definition 36 *We define* c_2Epatricia_2EPTREE__OF__NUMSET *to be* $\lambda V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2$

Let $c\_2Epatricia\_2EIS\_\_PTREE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EIS\_\_PTREE\ A\_27a \in (2^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \tag{21}$$

Let $c\_2Elist\_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EFILTER\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \tag{22}$$

Definition 37 *We define* c_2Esorting_2EPERM *to be* $\lambda A\_27a : \iota.\lambda V0L1 \in (ty\_2Elist\_2Elist\ A\_27a).\lambda V1L2$

Assume the following.

$$True \tag{23}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{27}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{31}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{32}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{33}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ A\_27a\ A\_27b)\ V0x)\ V1y) = V0x))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1x \in A\_27b.(\forall V2y \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EC}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ V1x)\ V2y) = (ap\ (ap\ V0f\ V2y)\ V1x))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0f \in ((A\_27b^{A\_27a})^{A\_27b}).(\forall V1e \in A\_27b.((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b)\ V0f)\ V1e)\ (c\_2Elist\_2ENIL\ A\_27a)) = V1e))) \wedge (\forall V2f \in ((A\_27b^{A\_27a})^{A\_27b}).(\forall V3e \in A\_27b.(\forall V4x \in A\_27a.(\forall V5l \in (ty\_2Elist\_2Elist\ A\_27a). ((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b)\ V2f)\ V3e)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V4x)\ V5l)) = (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b)\ V2f)\ (ap\ (ap\ V2f\ V3e)\ V4x))\ V5l))))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)))) \Leftrightarrow False)) \wedge (\forall V1x \in A\_27a.(\forall V2h \in A\_27a.(\forall V3t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V3t)))) \Leftrightarrow ((V1x = V2h) \vee (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V3t))))))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((p\ (ap\ (c\_2Elist\_2EALL\_\_DISTINCT\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a))) \Leftrightarrow True) \wedge (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (c\_2Elist\_2EALL\_\_DISTINCT\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t))) \Leftrightarrow ((\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0h)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V1t)))) \wedge (p\ (ap\ (c\_2Elist\_2EALL\_\_DISTINCT\ A\_27a)\ V1t))))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \Rightarrow (\forall V1x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ (ap\ (\mathsf{c\_2Elist\_2ESET\_\_TO\_\_LIST}\ A\_27a)\ V0s)))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ V0s)))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Elist\_2ESET\_\_TO\_\_LIST}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0x)\ (c\_2Elist\_2ENIL\ A\_27a)))) \quad (41)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \Rightarrow (p\ (ap\ (c\_2Elist\_2EALL\_\_DISTINCT\ A\_27a)\ (ap\ (\mathsf{c\_2Elist\_2ESET\_\_TO\_\_LIST}\ A\_27a)\ V0s))))) \quad (42)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1t \in (ty\_2Epatricia\_2Eptree\ A\_27a).((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ A\_27a)\ V1t)) \Rightarrow (p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ A\_27b)\ (ap\ (ap\ (c\_2Epatricia\_2ETRANSFORM\ A\_27b\ A\_27a)\ V0f)\ V1t)))))) \quad (43)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1t \in (ty\_2Epatricia\_2Eptree\ A\_27a).(\forall V2k \in ty\_2Enum\_2Enum.(\forall V3d \in A\_27a.((ap\ (ap\ (c\_2Epatricia\_2ETRANSFORM\ A\_27b\ A\_27a)\ V0f)\ (ap\ (ap\ (c\_2Epatricia\_2EADD\ A\_27a)\ V1t)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ A\_27a)\ V2k)\ V3d))) = (ap\ (ap\ (c\_2Epatricia\_2EADD\ A\_27b)\ (ap\ (ap\ (c\_2Epatricia\_2ETRANSFORM\ A\_27b\ A\_27a)\ V0f)\ V1t))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ A\_27b)\ V2k)\ (ap\ V0f\ V3d)))))))) \quad (44)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1t \in (ty\_2Epatricia\_2Eptree\ A\_27a).((ap\ (c\_2Epatricia\_2ETRAVERSE\ A\_27b)\ (ap\ (ap\ (c\_2Epatricia\_2ETRANSFORM\ A\_27b\ A\_27a)\ V0f)\ V1t)) = (ap\ (c\_2Epatricia\_2ETRAVERSE\ A\_27a)\ V1t)))) \quad (45)$$

Assume the following.

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V0t)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V1n)\ (ap\ \mathsf{c\_2Epatricia\_2ENUMSET\_\_OF\_\_PTREE}\ V0t))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EIN\_\_PTREE}\ V1n)\ V0t)))))) \quad (46)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in (ty\_2Epatricia\_2Eptree\ A\_27a).((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ A\_27a)\ V0t)) \Rightarrow (p\ (ap\ (c\_2Elist\_2EALL\_\_DISTINCT\ ty\_2Enum\_2Enum)\ (ap\ (c\_2Epatricia\_2ETRAVERSE\ A\_27a)\ V0t)))))\tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).(((p\ (ap\ (c\_2Elist\_2EALL\_\_DISTINCT\ A\_27a)\ V0l1)) \wedge ((p\ (ap\ (c\_2Elist\_2EALL\_\_DISTINCT\ A\_27a)\ V1l2)) \wedge (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V0l1))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V1l2))))))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0l1)\ V1l2)))))\tag{48}$$

Assume the following.

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1k \in ty\_2Enum\_2Enum.((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V0t)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V1k)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ ty\_2Enum\_2Enum)\ (ap\ (c\_2Epatricia\_2ETRAVERSE\ ty\_2Eone\_2Eone)\ V0t)))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V1k)\ (ap\ \mathsf{c\_2Epatricia\_2ENUMSET\_\_OF\_\_PTREE}\ V0t)))))))\tag{49}$$

Assume the following.

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1x \in ty\_2Enum\_2Enum.((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V0t)) \Rightarrow (p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EINSERT\_\_PTREE}\ V1x)\ V0t))))))\tag{50}$$

Assume the following.

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Enum\_2Enum)\ (ap\ \mathsf{c\_2Epatricia\_2ENUMSET\_\_OF\_\_PTREE}\ V0t))))\tag{51}$$

Assume the following.

$$(\forall V0v \in ty\_2Eone\_2Eone.(\forall V1t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V2n \in ty\_2Enum\_2Enum.((ap\ (ap\ (c\_2Epatricia\_2EADD\ ty\_2Eone\_2Eone)\ V1t)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ ty\_2Eone\_2Eone)\ V2n)\ V0v)) = (ap\ (ap\ \mathsf{c\_2Epatricia\_2EINSERT\_\_PTREE}\ V2n)\ V1t)))))\tag{52}$$

Assume the following.

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1s \in (2^{ty\_2Enum\_2Enum}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Enum\_2Enum)\ V1s))\Rightarrow((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V0t))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ ty\_2Enum\_2Enum)\ (ap\ (c\_2Epatricia\_2ETRAVERSE\ ty\_2Eone\_2Eone)\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ ty\_2Enum\_2Enum\ (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone))\ (ap\ (\mathsf{c\_2Ecombin\_2EC}\ ty\_2Enum\_2Enum\ (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone)\ (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone))\ \mathsf{c\_2Epatricia\_2EINSERT\_\_PTREE}))\ V0t)\ (ap\ (\mathsf{c\_2Elist\_2ESET\_\_TO\_\_LIST}\ ty\_2Enum\_2Enum)\ V1s))))\ (ap\ (\mathsf{c\_2Elist\_2ESET\_\_TO\_\_LIST}\ ty\_2Enum\_2Enum)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ ty\_2Enum\_2Enum)\ (ap\ \mathsf{c\_2Epatricia\_2ENUMSET\_\_OF\_\_PTREE}\ V0t))\ V1s))))))))) \tag{53}$$

Assume the following.

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1s \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V0t))\wedge(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Enum\_2Enum)\ V1s)))\Rightarrow((ap\ \mathsf{c\_2Epatricia\_2ENUMSET\_\_OF\_\_PTREE}\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EPTREE\_\_OF\_\_NUMSET}\ V0t)\ V1s)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ ty\_2Enum\_2Enum)\ (ap\ \mathsf{c\_2Epatricia\_2ENUMSET\_\_OF\_\_PTREE}\ V0t))\ V1s))))) \tag{54}$$

Assume the following.

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).((ap\ (ap\ \mathsf{c\_2Epatricia\_2EPTREE\_\_OF\_\_NUMSET}\ V0t)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Enum\_2Enum)) = V0t)) \tag{55}$$

Assume the following.

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1m \in ty\_2Enum\_2Enum.(\forall V2n \in ty\_2Enum\_2Enum.((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V0t))\Rightarrow((p\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EIN\_\_PTREE}\ V2n)\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EINSERT\_\_PTREE}\ V1m)\ V0t)))\Leftrightarrow((V1m = V2n)\vee(p\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EIN\_\_PTREE}\ V2n)\ V0t)))))))) \tag{56}$$

Assume the following.

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1s \in (2^{ty\_2Enum\_2Enum}).(\forall V2x \in ty\_2Enum\_2Enum.(((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V0t))\wedge(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Enum\_2Enum)\ V1s)))\Rightarrow((ap\ (ap\ \mathsf{c\_2Epatricia\_2EPTREE\_\_OF\_\_NUMSET}\ V0t)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Enum\_2Enum)\ V2x)\ V1s)) = (ap\ (ap\ \mathsf{c\_2Epatricia\_2EINSERT\_\_PTREE}\ V2x)\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EPTREE\_\_OF\_\_NUMSET}\ V0t)\ V1s))))))) \tag{57}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))) \tag{58}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V0x)\ V1s))) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V1s))))) \tag{59}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{60}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{61}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{62}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{63}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{64}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \tag{65}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{66}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{67}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{68}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{69}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{70}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{71}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{72}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{73}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{74}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1y \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2z \in (ty\_2Elist\_2Elist\ A\_27a).(((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0x)\ V1y))\wedge(p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V1y)\ V2z)))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0x)\ V2z)))))) \tag{75}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t \in (ty\_2Epatricia\_2Eptree\ A\_27a).(\forall V1k \in ty\_2Enum\_2Enum.(\forall V2d \in A\_27a.(((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ A\_27a)\ V0t))\wedge(\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V1k)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ ty\_2Enum\_2Enum)\ (ap\ (c\_2Epatricia\_2ETRAVERSE\ A\_27a)\ V0t))))))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ ty\_2Enum\_2Enum)\ (ap\ (c\_2Epatricia\_2ETRAVERSE\ A\_27a)\ (ap\ (ap\ (c\_2Epatricia\_2EADD\ A\_27a)\ V0t)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ A\_27a)\ V1k)\ V2d))))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ty\_2Enum\_2Enum)\ V1k)\ (ap\ (c\_2Epatricia\_2ETRAVERSE\ A\_27a)\ V0t))))))))$$