

thm_2Epatricia_2Eperm_DELETE_PTREE
 (TMJSNovBYdSSAS-
 dqQNQ8cd3pEDUbVMC34rf)

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Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 1 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 2 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2E3F to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E40\ A\ 27a))))$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{2}$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty\ A.\lambda 27a \Rightarrow c_2Elist_2ENIL\ A.\lambda 27a \in (ty_2Elist_2Elist\ A.\lambda 27a) \tag{3}$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty\ A.\lambda 27a \Rightarrow c_2Elist_2ECONS\ A.\lambda 27a \in (((ty_2Elist_2Elist\ A.\lambda 27a)^{(ty_2Elist_2Elist\ A.\lambda 27a)})^{A.\lambda 27a}) \tag{4}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 6 We define c_2Ebool_2E21 to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a}))))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (5)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (6)$$

Definition 8 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS_sum A_27a A_27b) V0e)$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (7)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (8)$$

Definition 9 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap (c_2Eoption_2Eoption_ABS A_27a) V0x)$

Definition 10 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone.))$

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 13 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS_sum A_27a A_27b) V0e)$

Definition 14 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap (c_2Eoption_2Eoption_ABS A_27a) V0t)$

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.))$

Let $ty_2Epatricia_2Eptree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Epatricia_2Eptree A0) \quad (9)$$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty ty_2Eenum_2Eenum \quad (10)$$

Let $c_2Epatricia_2EREMOVE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Epatricia_2EREMOVE A_27a \in ((ty_2Epatricia_2Eptree A_27a)^{(ty_2Eenum_2Eenum)}(ty_2Epatricia_2Eptree A_27a)) \quad (11)$$

Let $c_2Epatricia_2EPEEK : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Epatricia_2EPEEK\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{ty_2Enum_2Enum})^{(ty_2Epatricia_2Eptree\ A_27a)}) \quad (12)$$

Let $c_2Eoption_2EIS_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2EIS_SOME\ A_27a \in (2^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (13)$$

Let $c_2Epatricia_2ETRAVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Epatricia_2ETRAVERSE\ A_27a \in ((ty_2Elist_2Elist\ ty_2Enum_2Enum)^{(ty_2Epatricia_2Eptree\ A_27a)}) \quad (14)$$

Let $c_2Epatricia_2EIS_PTREE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Epatricia_2EIS_PTREE\ A_27a \in (2^{(ty_2Epatricia_2Eptree\ A_27a)}) \quad (15)$$

Let $c_2Elist_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EFILTER\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})}) \quad (16)$$

Definition 16 We define $c_2Esorting_2Eperm$ to be $\lambda A_27a : \iota.\lambda V0L1 \in (ty_2Elist_2Elist\ A_27a).\lambda V1L2$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in (2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)} \quad (17)$$

Definition 17 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Let $c_2Elist_2EALL_DISTINCT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EALL_DISTINCT\ A_27a \in (2^{(ty_2Elist_2Elist\ A_27a)}) \quad (18)$$

Definition 18 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Assume the following.

$$True) \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge ((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0t1 \in A.27a. (\forall V1t2 \in \\ A.27a. (((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ A.27a)\ c.2Ebool.2ET)\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ A.27a)\ c.2Ebool.2EF) \\ V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (\\ (p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \quad (33)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (34)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in \\ 2. (((p\ V0x) \Leftrightarrow (p\ V1x.27)) \wedge ((p\ V1x.27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y.27)))))) \Rightarrow \\ (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x.27) \Rightarrow (p\ V3y.27)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1L \in \\ (ty.2Elist.2Elist\ A.27a). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN \\ A.27a)\ V2x)\ (ap\ (c.2Elist.2ELIST_TO_SET\ A.27a)\ (ap\ (ap\ (c.2Elist.2EFILTER \\ A.27a)\ V0P)\ V1L)))) \Leftrightarrow ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ (ap\ (c.2Ebool.2EIN \\ A.27a)\ V2x)\ (ap\ (c.2Elist.2ELIST_TO_SET\ A.27a)\ V1L))))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty.2Elist.2Elist \\ A.27a). ((p\ (ap\ (c.2Elist.2EALL_DISTINCT\ A.27a)\ V0l)) \Leftrightarrow (\forall V1x \in \\ A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1x)\ (ap\ (c.2Elist.2ELIST_TO_SET \\ A.27a)\ V0l))) \Rightarrow ((ap\ (ap\ (c.2Elist.2EFILTER\ A.27a)\ (ap\ (c.2Emin.2E.3D \\ A.27a)\ V1x))\ V0l) = (ap\ (ap\ (c.2Elist.2ECONS\ A.27a)\ V1x)\ (c.2Elist.2ENIL \\ A.27a)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0x \in A.27a. ((p\ (ap\ (c.2Eoption.2EIS_SOME \\ A.27a)\ (ap\ (c.2Eoption.2ESOME\ A.27a)\ V0x))) \Leftrightarrow True)) \wedge ((p\ (ap\ (c.2Eoption.2EIS_SOME \\ A.27a)\ (c.2Eoption.2ENONE\ A.27a))) \Leftrightarrow False)) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in (ty_2Epatricia_2Eptree \\ A_27a).(\forall V1k \in ty_2Enum_2Enum.((p\ (ap\ (c_2Epatricia_2EIS_PTREE \\ A_27a)\ V0t)) \Rightarrow (p\ (ap\ (c_2Epatricia_2EIS_PTREE\ A_27a)\ (ap\ (ap\ (\\ c_2Epatricia_2EREMOVE\ A_27a)\ V0t)\ V1k)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in (ty_2Epatricia_2Eptree \\ A_27a).(\forall V1k \in ty_2Enum_2Enum.(\forall V2j \in ty_2Enum_2Enum. \\ ((p\ (ap\ (c_2Epatricia_2EIS_PTREE\ A_27a)\ V0t)) \Rightarrow ((ap\ (ap\ (c_2Epatricia_2EPEEK \\ A_27a)\ (ap\ (ap\ (c_2Epatricia_2EREMOVE\ A_27a)\ V0t)\ V1k))\ V2j) = (\\ ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a))\ (ap \\ (ap\ (c_2Emin_2E_3D\ ty_2Enum_2Enum)\ V1k)\ V2j))\ (c_2Eoption_2ENONE \\ A_27a))\ (ap\ (ap\ (c_2Epatricia_2EPEEK\ A_27a)\ V0t)\ V2j)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in (ty_2Epatricia_2Eptree \\ A_27a).(\forall V1k \in ty_2Enum_2Enum.((p\ (ap\ (c_2Epatricia_2EIS_PTREE \\ A_27a)\ V0t)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ ty_2Enum_2Enum)\ V1k)\ (\\ ap\ (c_2Elist_2ELIST_TO_SET\ ty_2Enum_2Enum)\ (ap\ (c_2Epatricia_2ETRAVERSE \\ A_27a)\ V0t)))) \Leftrightarrow (p\ (ap\ (c_2Eoption_2EIS_SOME\ A_27a)\ (ap\ (ap\ (c_2Epatricia_2EPEEK \\ A_27a)\ V0t)\ V1k)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in (ty_2Epatricia_2Eptree \\ A_27a).((p\ (ap\ (c_2Epatricia_2EIS_PTREE\ A_27a)\ V0t)) \Rightarrow (p\ (ap \\ (c_2Elist_2EALL_DISTINCT\ ty_2Enum_2Enum)\ (ap\ (c_2Epatricia_2ETRAVERSE \\ A_27a)\ V0t)))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l1 \in (ty_2Elist_2Elist \\ A_27a).(\forall V1l2 \in (ty_2Elist_2Elist\ A_27a).(((p\ (ap\ (c_2Elist_2EALL_DISTINCT \\ A_27a)\ V0l1)) \wedge ((p\ (ap\ (c_2Elist_2EALL_DISTINCT\ A_27a)\ V1l2)) \wedge \\ (\forall V2x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ (ap\ (\\ c_2Elist_2ELIST_TO_SET\ A_27a)\ V0l1))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ A_27a)\ V2x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ V1l2)))))) \Rightarrow \\ (p\ (ap\ (ap\ (c_2Esorting_2Eperm\ A_27a)\ V0l1)\ V1l2)))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & (2^{A.27a}).(\forall V2l \in (ty.2Elist.2Elist\ A.27a).((ap\ (ap\ (c.2Elist.2EFILTER \\ & A.27a)\ V0P)\ (ap\ (ap\ (c.2Elist.2EFILTER\ A.27a)\ V1Q)\ V2l)) = (ap\ (ap \\ & (c.2Elist.2EFILTER\ A.27a)\ (\lambda V3x \in A.27a.(ap\ (ap\ c.2Ebool.2E_2F_5C \\ & (ap\ V0P\ V3x))\ (ap\ V1Q\ V3x))))\ V2l)))))) \end{aligned} \quad (44)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A) \vee (p\ V1B)))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(\neg(p\ V0A)) \vee (p\ V1B)))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (48)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (49)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & (p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ & ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (52)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q) \vee ((p V2r) \vee \neg(p V0p)))))))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p)))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q)))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V0p)))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q)))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (59)$$

Theorem 1

$$(\forall V0t \in (ty_2Epatricia_2Eptree\ ty_2Eone_2Eone). (\forall V1k \in ty_2Enum_2Enum. ((p (ap (c_2Epatricia_2EIS_PTREE\ ty_2Eone_2Eone) V0t)) \wedge (p (ap (ap (c_2Ebool_2EIN\ ty_2Enum_2Enum) V1k) (ap (c_2Elist_2ELIST_TO_SET\ ty_2Enum_2Enum) (ap (c_2Epatricia_2ETRAVERSE\ ty_2Eone_2Eone) V0t)))))) \Rightarrow (p (ap (ap (c_2Esorting_2Eperm\ ty_2Enum_2Enum) (ap (c_2Epatricia_2ETRAVERSE\ ty_2Eone_2Eone) (ap (ap (c_2Epatricia_2EREMOVE\ ty_2Eone_2Eone) V0t) V1k))) (ap (ap (c_2Elist_2EFILTER\ ty_2Enum_2Enum) (\lambda V2x \in ty_2Enum_2Enum. (ap c_2Ebool_2E7E (ap (ap (c_2Emin_2E3D\ ty_2Enum_2Enum) V2x) V1k)))) (ap (c_2Epatricia_2ETRAVERSE\ ty_2Eone_2Eone) V0t))))))))))$$