thm_2Epatricia_2EPERM__INSERT__PTREE (TMdhs5uzqthSXBnxBFsRvA2qyjw7yHxUMi2)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 4 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Definition 5 *We define* c_2Ecombin_2EC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27c}$

Definition 6 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 7 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

Definition 8 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (c\_2Ecombin\_2ES\ A\_27a\ (A\_27a^{A\_27a})\ A\_$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (1)$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (2)$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (3)$$

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \quad (4)$$

Let $c\_2Epred\_\_set\_2ECHOICE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epred\_\_set\_2ECHOICE\ A\_27a \in (A\_27a^{(2^{A\_27a})}) \quad (5)$$

Definition 9 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 11 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 12 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 13 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 14 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{6}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{7}$$

Definition 15 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{8}$$

Definition 16 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 17 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 18 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Definition 19 *We define* c_2Epred__set_2EDELETE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap\ (a$

Definition 20 *We define* c_2Epred__set_2EREST *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2}$

Definition 21 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 22 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 23 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ (2$

Definition 24 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 25 *We define* c_2Erelation_2EWF *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap$ (c_2Ebool_2E_21

Definition 26 *We define* c_2Erelation_2ERESTRICT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1$

Definition 27 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 28 *We define* c_2Erelation_2Eapprox *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 29 *We define* c_2Erelation_2Ethe__fun *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1$

Definition 30 *We define* c_2Erelation_2EWFREC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1$

Definition 31 *We define* c_2Elist_2ESET__TO__LIST *to be* $\lambda A\_27a : \iota.(ap\ (ap$ (c_2Erelation_2EWFREC $(2^{A$

Let $c\_2Elist\_2EALL\_\_DISTINCT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EALL\_\_DISTINCT\ A\_27a \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{9}$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{10}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{11}$$

Let $ty\_2Epatricia\_2Eptree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Epatricia\_2Eptree\ A0) \tag{12}$$

Let $c\_2Epatricia\_2ETRAVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2ETRAVERSE\ A\_27a \in ((ty\_2Elist\_2Elist\ ty\_2Enum\_2Enum)^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \tag{13}$$

Let $c\_2Elist\_2ELIST\_\_TO\_\_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{14}$$

Definition 32 *We define* c_2Epatricia_2ENUMSET__OF__PTREE *to be* $\lambda V0t \in (ty\_2Epatricia\_2Eptree\ ty\_$

Definition 33 *We define* c_2Eone_2Eone *to be* $(ap$ (c_2Emin_2E_40 $ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_$

Let $c\_2Epatricia\_2EADD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EADD\ A\_27a \in (((ty\_2Epatricia\_2Eptree\ A\_27a)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ A\_27a)})^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \tag{15}$$

Definition 34 *We define* c_2Epatricia_2EINSERT__PTREE *to be* $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1t \in (ty\_2E$

Let $c\_2Epatricia\_2EIS\_\_PTREE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EIS\_\_PTREE\ A\_27a \in (2^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \tag{16}$$

Definition 35 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{17}$$

Let $c\_2Elist\_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EREVERSE\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{18}$$

Let $c\_2Elist\_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EFOLDR\ A\_27a\ A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27b})^{((A\_27b^{A\_27b})^{A\_27a})}) \tag{19}$$

Let $c\_2Elist\_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27b})^{((A\_27b^{A\_27a})^{A\_27b})}) \tag{20}$$

Let $c\_2Elist\_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EFILTER\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \tag{21}$$

Definition 36 *We define* c_2Esorting_2EPERM *to be* $\lambda A\_27a : \iota.\lambda V0L1 \in (ty\_2Elist\_2Elist\ A\_27a).\lambda V1L2$

Assume the following.

$$True \tag{22}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{26}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{31}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \tag{32}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{33}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}\forall A\_27c.\ nonempty\ A\_27c{\Rightarrow}(\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1x \in A\_27b.(\forall V2y \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EC}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ V1x)\ V2y) = (ap\ (ap\ V0f\ V2y)\ V1x))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}(\forall V0h \in A\_27b.(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27b).(((ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)){\wedge}((ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V0h)\ V1t)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27b)\ V0h)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27b)\ V1t)))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}((\forall V0f \in ((A\_27b^{A\_27b})^{A\_27a}).(\forall V1e \in A\_27b.((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDR\ A\_27a\ A\_27b)\ V0f)\ V1e)\ (c\_2Elist\_2ENIL\ A\_27a)) = V1e))){\wedge}(\forall V2f \in ((A\_27b^{A\_27b})^{A\_27a}).(\forall V3e \in A\_27b.(\forall V4x \in A\_27a.(\forall V5l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDR\ A\_27a\ A\_27b)\ V2f)\ V3e)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V4x)\ V5l)) = (ap\ (ap\ V2f\ V4x)\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDR\ A\_27a\ A\_27b)\ V2f)\ V3e)\ V5l)))))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}).(((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))){\wedge}(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ V0P\ V1t)){\Rightarrow}(\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t))))))){\Rightarrow}(\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ V0P\ V3l))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(((ap\ (c\_2Elist\_2EREVERSE\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) = (c\_2Elist\_2ENIL\ A\_27a)){\wedge}(\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2EREVERSE\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (c\_2Elist\_2EREVERSE\ A\_27a)\ V1t))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ (c\_2Elist\_2ENIL\ A\_27a))))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((p\ (ap\ (c\_2Elist\_2EALL\_\_DISTINCT\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a))) \Leftrightarrow True) \wedge (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (c\_2Elist\_2EALL\_\_DISTINCT\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t))) \Leftrightarrow ((\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0h)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V1t)))) \wedge (p\ (ap\ (c\_2Elist\_2EALL\_\_DISTINCT\ A\_27a)\ V1t))))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V0l)))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \Rightarrow ((ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ (ap\ (\mathsf{c\_2Elist\_2ESET\_\_TO\_\_LIST}\ A\_27a)\ V0s)) = V0s))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \Rightarrow (\forall V1x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ (ap\ (\mathsf{c\_2Elist\_2ESET\_\_TO\_\_LIST}\ A\_27a)\ V0s)))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1x)\ V0s)))))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \Rightarrow (p\ (ap\ (c\_2Elist\_2EALL\_\_DISTINCT\ A\_27a)\ (ap\ (\mathsf{c\_2Elist\_2ESET\_\_TO\_\_LIST}\ A\_27a)\ V0s))))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in (ty\_2Epatricia\_2Eptree\ A\_27a).((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ A\_27a)\ V0t)) \Rightarrow (p\ (ap\ (c\_2Elist\_2EALL\_\_DISTINCT\ ty\_2Enum\_2Enum)\ (ap\ (c\_2Epatricia\_2ETRAVERSE\ A\_27a)\ V0t))))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).(((p\ (ap\ (c\_2Elist\_2EALL\_\_DISTINCT\ A\_27a)\ V0l1)) \wedge ((p\ (ap\ (c\_2Elist\_2EALL\_\_DISTINCT\ A\_27a)\ V1l2)) \wedge (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V0l1))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V1l2))))))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0l1)\ V1l2))))) \tag{47}$$

Assume the following.

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1k \in ty\_2Enum\_2Enum.((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V0t)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V1k)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ ty\_2Enum\_2Enum)\ (ap\ (c\_2Epatricia\_2ETRAVERSE\ ty\_2Eone\_2Eone)\ V0t)))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V1k)\ (ap\ \mathsf{c\_2Epatricia\_2ENUMSET\_\_OF\_\_PTREE}\ V0t))))))) \tag{48}$$

Assume the following.

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1x \in ty\_2Enum\_2Enum.((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V0t)) \Rightarrow (p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EINSERT\_\_PTREE}\ V1x)\ V0t)))))) \tag{49}$$

Assume the following.

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Enum\_2Enum)\ (ap\ \mathsf{c\_2Epatricia\_2ENUMSET\_\_OF\_\_PTREE}\ V0t)))) \tag{50}$$

Assume the following.

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1x \in ty\_2Enum\_2Enum.(\forall V2h \in ty\_2Enum\_2Enum.((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V0t)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V1x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ ty\_2Enum\_2Enum)\ (ap\ (c\_2Epatricia\_2ETRAVERSE\ ty\_2Eone\_2Eone)\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EINSERT\_\_PTREE}\ V2h)\ V0t))))) \Leftrightarrow ((V1x = V2h) \vee ((\neg(V1x = V2h)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V1x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ ty\_2Enum\_2Enum)\ (ap\ (c\_2Epatricia\_2ETRAVERSE\ ty\_2Eone\_2Eone)\ V0t)))))))))) \tag{51}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{52}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0s \in (2^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))\ V0s) = V0s)) \wedge (\forall V1s \in (2^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ V1s)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)) = V1s))) \tag{53}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = V1y) \vee (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V2s))))))) \tag{54}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1s \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V0x)\ V1s))) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V1s))))) \tag{55}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \wedge (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V1t)))))) \tag{56}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((A\_27b^{A\_27b})^{A\_27a}).(\forall V1e \in A\_27b.(\forall V2l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3l2 \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDR\ A\_27a\ A\_27b)\ V0f)\ V1e)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V2l1)\ V3l2)) = (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDR\ A\_27a\ A\_27b)\ V0f)\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDR\ A\_27a\ A\_27b)\ V0f)\ V1e)\ V3l2))\ V2l1)))))) \tag{57}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((A\_27a^{A\_27b})^{A\_27a}).(\forall V1e \in A\_27a.(\forall V2l \in (ty\_2Elist\_2Elist\ A\_27b).((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27b\ A\_27a)\ V0f)\ V1e)\ V2l) = (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDR\ A\_27b\ A\_27a)\ (\lambda V3x \in A\_27b.(\lambda V4y \in A\_27a.(ap\ (ap\ V0f\ V4y)\ V3x))))\ V1e)\ (ap\ (c\_2Elist\_2EREVERSE\ A\_27b)\ V2l)))))) \tag{58}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{59}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{60}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{61}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{62}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{63}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \tag{64}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{65}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \tag{66}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{67}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{68}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{69}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{70}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{71}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{72}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{73}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1l2 \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ A\_27a)\ V0l1)\ V1l2))\Rightarrow(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V0l1)))\Leftrightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V1l2)))))))) \tag{74}$$

Theorem 1

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1s \in (2^{ty\_2Enum\_2Enum}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Enum\_2Enum)\ V1s))\Rightarrow((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V0t))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Esorting\_2EPERM}\ ty\_2Enum\_2Enum)\ (ap\ (c\_2Epatricia\_2ETRAVERSE\ ty\_2Eone\_2Eone)\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ ty\_2Enum\_2Enum\ (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone))\ (ap\ (\mathsf{c\_2Ecombin\_2EC}\ ty\_2Enum\_2Enum\ (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone)\ (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone))\ \mathsf{c\_2Epatricia\_2EINSERT\_\_PTREE}))\ V0t)\ (ap\ (\mathsf{c\_2Elist\_2ESET\_\_TO\_\_LIST}\ ty\_2Enum\_2Enum)\ V1s))))\ (ap\ (\mathsf{c\_2Elist\_2ESET\_\_TO\_\_LIST}\ ty\_2Enum\_2Enum)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ ty\_2Enum\_2Enum)\ (ap\ \mathsf{c\_2Epatricia\_2ENUMSET\_\_OF\_\_PTREE}\ V0t))\ V1s))))))))$$