

# thm\_2Epatricia\_2EPERM\_REMOVE (TMQXF8DjthfipLUQtfpDxnsouRKFdBExTPk)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

Let  $ty\_2Epatricia\_2Eptree : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Epatricia\_2Eptree A0) \quad (1)$$

Let  $c\_2Epatricia\_2ETRANSFORM : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epatricia\_2ETRANSFORM A\_27a A\_27b \in (((ty\_2Epatricia\_2Eptree A\_27a)^{(ty\_2Epatricia\_2Eptree A\_27b)})^{(A\_27a^{A\_27b})}) \quad (2)$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (3)$$

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF$

Let  $c\_2Elist\_2EFILTER : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EFILTER A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{(2^{A\_27a})}) \quad (4)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (5)$$

Let  $c\_2Epatricia\_2EREMOVE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EREMOVE\ A\_27a \in ( ((ty\_2Epatricia\_2Eptree\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Epatricia\_2Eptree\ A\_27a)} ) \quad (6)$$

**Definition 7** We define  $c\_2Esorting\_2EPERM$  to be  $\lambda A\_27a : \iota. \lambda V0L1 \in (ty\_2Elist\_2Elist\ A\_27a). \lambda V1L2 \in$

Let  $c\_2Epatricia\_2ETRAVERSE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2ETRAVERSE\ A\_27a \in ((ty\_2Elist\_2Elist\ ty\_2Enum\_2Enum)^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \quad (7)$$

Let  $c\_2Elist\_2ELIST\_TO\_SET : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELIST\_TO\_SET\ A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (8)$$

**Definition 8** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap\ V1f\ V0x)))$

Let  $c\_2Epatricia\_2EIS\_PTREE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EIS\_PTREE\ A\_27a \in (2^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \quad (9)$$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (10)$$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (13)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (14)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (17)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1t \in (ty\_2Epatricia\_2Eptree\ A\_27a). ((p\ (ap\ (c\_2Epatricia\_2EIS\_PTREE\ A\_27a)\ V1t)) \Rightarrow (p\ (ap\ (c\_2Epatricia\_2EIS\_PTREE\ A\_27b)\ (ap\ (ap\ (c\_2Epatricia\_2ETRANSFORM\ A\_27b\ A\_27a)\ V0f)\ V1t)))))) \quad (19)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1t \in (ty\_2Epatricia\_2Eptree\ A\_27a). (\forall V2k \in ty\_2Enum\_2Enum. ((ap\ (ap\ (c\_2Epatricia\_2ETRANSFORM\ A\_27b\ A\_27a)\ V0f)\ (ap\ (ap\ (c\_2Epatricia\_2EREMOVE\ A\_27a)\ V1t)\ V2k)) = (ap\ (ap\ (c\_2Epatricia\_2EREMOVE\ A\_27b)\ (ap\ (ap\ (c\_2Epatricia\_2ETRANSFORM\ A\_27b\ A\_27a)\ V0f)\ V1t))\ V2k)))))) \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1t \in (ty\_2Epatricia\_2Eptree\ A\_27a). ((ap\ (c\_2Epatricia\_2ETRAVERSE\ A\_27b)\ (ap\ (ap\ (c\_2Epatricia\_2ETRANSFORM\ A\_27b\ A\_27a)\ V0f)\ V1t)) = (ap\ (c\_2Epatricia\_2ETRAVERSE\ A\_27a)\ V1t)))) \quad (21)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1k \in \\
& \quad ty\_2Enum\_2Enum.(((p (ap (c\_2Epatricia\_2EIS\_PTREE\ ty\_2Eone\_2Eone) \\
V0t)) \wedge (p (ap (ap (c\_2Ebool\_2EIN\ ty\_2Enum\_2Enum) V1k) (ap (c\_2Elist\_2ELIST\_TO\_SET \\
\quad ty\_2Enum\_2Enum) (ap (c\_2Epatricia\_2ETRAVERSE\ ty\_2Eone\_2Eone) \\
\quad V0t)))))) \Rightarrow (p (ap (ap (c\_2Esorting\_2EPERM\ ty\_2Enum\_2Enum) (ap ( \\
c\_2Epatricia\_2ETRAVERSE\ ty\_2Eone\_2Eone) (ap (ap (c\_2Epatricia\_2EREMOVE \\
\quad ty\_2Eone\_2Eone) V0t) V1k))) (ap (ap (c\_2Elist\_2EFILTER\ ty\_2Enum\_2Enum) \\
\quad (\lambda V2x \in ty\_2Enum\_2Enum.(ap c\_2Ebool\_2E\_7E (ap (ap (c\_2Emin\_2E\_3D \\
\quad ty\_2Enum\_2Enum) V2x) V1k)))) (ap (c\_2Epatricia\_2ETRAVERSE\ ty\_2Eone\_2Eone) \\
\quad V0t)))))))))
\end{aligned} \tag{22}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in (ty\_2Epatricia\_2Eptree \\
& \quad A\_27a).(\forall V1k \in ty\_2Enum\_2Enum.(((p (ap (c\_2Epatricia\_2EIS\_PTREE \\
& \quad A\_27a) V0t)) \wedge (p (ap (ap (c\_2Ebool\_2EIN\ ty\_2Enum\_2Enum) V1k) (ap \\
(c\_2Elist\_2ELIST\_TO\_SET\ ty\_2Enum\_2Enum) (ap (c\_2Epatricia\_2ETRAVERSE \\
\quad A\_27a) V0t)))))) \Rightarrow (p (ap (ap (c\_2Esorting\_2EPERM\ ty\_2Enum\_2Enum) \\
(ap (c\_2Epatricia\_2ETRAVERSE\ A\_27a) (ap (ap (c\_2Epatricia\_2EREMOVE \\
\quad A\_27a) V0t) V1k))) (ap (ap (c\_2Elist\_2EFILTER\ ty\_2Enum\_2Enum) \\
\quad (\lambda V2x \in ty\_2Enum\_2Enum.(ap c\_2Ebool\_2E\_7E (ap (ap (c\_2Emin\_2E\_3D \\
\quad ty\_2Enum\_2Enum) V2x) V1k)))) (ap (c\_2Epatricia\_2ETRAVERSE\ A\_27a) \\
\quad V0t)))))))))
\end{aligned}$$