

thm_2Epatricia_2EPTREE_OF_NUMSET_EMPTY
 (TMarcYVB-
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October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c_2Elist_2ECONS A.27a \in (((ty_2Elist_2Elist A.27a)^{(ty_2Elist_2Elist A.27a)})^{A.27a}) \quad (2)$$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c_2Elist_2ENIL A.27a \in (ty_2Elist_2Elist A.27a) \quad (3)$$

Definition 6 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A.27a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2EF)$.

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (4)$$

Let $c_2Epred_set_2ECHOICE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Epred_set_2ECHOICE\ A_27a \in (A_27a)^{(2^{A_27a})} \quad (5)$$

Definition 8 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 9 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Ebool_2E_5C_2F\ t1\ t2)\ V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (6)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b))^{((2^{A_27b})^{A_27a})} \quad (7)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2E_2C\ x\ y)\ V0x\ V1y)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \quad (8)$$

Definition 11 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Epred_set_2EINSERT\ x\ s)\ V0x\ V1s)$

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E\ V0t))$

Definition 13 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Epred_set_2EDIFF\ s\ t)\ V0s\ V1t)$

Definition 14 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1x \in A_27a. (ap\ (ap\ c_2Epred_set_2EDELETE\ s\ x)\ V0s\ V1x)$

Definition 15 We define $c_2Epred_set_2EREST$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap\ (ap\ c_2Epred_set_2EREST\ s)\ V0s)$

Definition 16 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x. x \in A \wedge P\ x)) \text{ of type } \iota \Rightarrow \iota.$

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (c_2Ebool_2ECOND\ t1\ t2\ V0t))))$

Definition 18 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap\ (c_2Ebool_2E_21\ 2)\ V0s)$

Definition 19 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x\ V1y))$

Definition 20 We define $c_Ecombin_2ES$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 21 We define $c_Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap (ap (c_Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Definition 22 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_Emin_2E_40 A_27a P))))$

Definition 23 We define $c_ERelation_2EWF$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). (ap (c_Ebool_2E_3F A_27a R))$

Definition 24 We define $c_ERelation_2ERESTRICT$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1g \in (A_27a^{A_27b}). (ap (ap (c_Ebool_2E_3F A_27a (lambda V0R \in (2^{A_27a}). (ap V0R (ap (c_Emin_2E_40 A_27a R)))))) A_27b f g))$

Definition 25 We define $c_ERelation_2ETC$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1a \in A_27a. \lambda V2b \in A_27a. (ap (ap (c_Emin_2E_40 A_27a (lambda V0R \in (2^{A_27a}). (ap V0R (ap (c_Emin_2E_40 A_27a R)))))) V1a V2b))$

Definition 26 We define $c_ERelation_2Eapprox$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M \in (A_27a^{A_27b}). (ap (ap (c_Emin_2E_40 A_27a (lambda V0R \in (2^{A_27a}). (ap V0R (ap (c_Emin_2E_40 A_27a R)))))) V1M))$

Definition 27 We define $c_ERelation_2Ethe_fun$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M \in (A_27a^{A_27b}). (ap (ap (c_Emin_2E_40 A_27a (lambda V0R \in (2^{A_27a}). (ap V0R (ap (c_Emin_2E_40 A_27a R)))))) V1M))$

Definition 28 We define $c_ERelation_2EWFREC$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M \in (A_27a^{A_27b}). (ap (ap (c_Emin_2E_40 A_27a (lambda V0R \in (2^{A_27a}). (ap V0R (ap (c_Emin_2E_40 A_27a R)))))) V1M))$

Definition 29 We define $c_Elist_2ESET_TO_LIST$ to be $\lambda A_27a : \iota. (ap (ap (c_ERelation_2EWFREC (2^{A_27a})^{A_27a}) A_27a))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{9}$$

Definition 30 We define c_Eone_2Eone to be $(ap (c_Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{10}$$

Let $ty_2Epatricia_2Eptree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Epatricia_2Eptree\ A0) \tag{11}$$

Let $c_2Epatricia_2EADD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Epatricia_2EADD\ A_27a \in (((ty_2Epatricia_2Eptree\ A_27a)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ A_27a)})^{(ty_2Epatricia_2Eptree\ A_27a)}) \tag{12}$$

Definition 31 We define $c_2Epatricia_2EINSERT_PTREE$ to be $\lambda V0n \in ty_2Enum_2Enum. \lambda V1t \in (ty_2Epatricia_2Eptree\ n)$

Definition 32 We define $c_Ecombin_2EC$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDL\ A_27a\ A_27b \in (((A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{((A_27b^{A_27a})^{A_27b})}) \tag{13}$$

Definition 33 We define $c_Epatricia_EPTREE_OF_NUMSET$ to be $\lambda V0t \in (ty_Epatricia_Eptree\ ty_Eone_Eone)$

Assume the following.

$$True \tag{14}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{15}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad (\forall V0f \in ((A_27b^{A_27a})^{A_27b}). (\forall V1e \in A_27b. ((ap\ (\\ & \quad ap\ (ap\ (c_Elist_EFOLDL\ A_27a\ A_27b)\ V0f)\ V1e)\ (c_Elist_ENIL \\ & \quad A_27a)) = V1e))) \wedge (\forall V2f \in ((A_27b^{A_27a})^{A_27b}). (\forall V3e \in \\ & \quad A_27b. (\forall V4x \in A_27a. (\forall V5l \in (ty_Elist_Elist\ A_27a). \\ & \quad ((ap\ (ap\ (ap\ (c_Elist_EFOLDL\ A_27a\ A_27b)\ V2f)\ V3e)\ (ap\ (ap\ (c_Elist_ECONS \\ & \quad A_27a)\ V4x)\ V5l)) = (ap\ (ap\ (ap\ (c_Elist_EFOLDL\ A_27a\ A_27b)\ V2f) \\ & \quad (ap\ (ap\ V2f\ V3e)\ V4x))\ V5l)))))) \end{aligned} \tag{16}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((ap\ (c_Elist_ESET_TO_LIST\ A_27a)\ (c_Epred_set_EEMPTY\ A_27a)) = (c_Elist_ENIL\ A_27a)) \tag{17}$$

Theorem 1

$$\begin{aligned} & (\forall V0t \in (ty_Epatricia_Eptree\ ty_Eone_Eone). ((ap\ (\\ & \quad ap\ c_Epatricia_EPTREE_OF_NUMSET\ V0t)\ (c_Epred_set_EEMPTY \\ & \quad ty_Eenum_Eenum)) = V0t)) \end{aligned}$$