thm_2Epatricia_2EPTREE__OF__NUMSET__NUMSET__OF__PTR (TM-NTabLej5B8ddFrPRjAB1Cd1n3CdAz2GaM)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{3}$$

Let $ty\_2Epatricia\_2Eptree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Epatricia\_2Eptree\ A0) \tag{4}$$

Let $c\_2Epatricia\_2ETRAVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2ETRAVERSE\ A\_27a \in ((ty\_2Elist\_2Elist\ ty\_2Enum\_2Enum)^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \tag{5}$$

Let $c\_2Elist\_2ELIST\_\_TO\_\_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{6}$$

Definition 5 *We define* c_2Epatricia_2ENUMSET__OF__PTREE *to be* $\lambda V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2$

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \tag{7}$$

Let $c\_2Epred\_\_set\_2ECHOICE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epred\_\_set\_2ECHOICE\ A\_27a \in (A\_27a^{(2^{A\_27a})}) \tag{8}$$

Definition 6 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 7 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 8 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 9 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{9}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{10}$$

Definition 11 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{11}$$

Definition 12 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 13 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 14 *We define* c_2Epred__set_2EDIFF *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c\_2$

Definition 15 *We define* c_2Epred__set_2EDELETE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1x \in A\_27a.(ap\ (a$

Definition 16 *We define* c_2Epred__set_2EREST *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2}$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{12}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{13}$$

Definition 17 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 18 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 19 *We define* c_2Epred__set_2EFINITE *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap$ (c_2Ebool_2E_21 (2

Definition 20 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 21 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 22 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap$ (c_2Ecombin_2ES $A\_27a\ (A\_27a^{A\_27a})\ A$

Definition 23 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 24 *We define* c_2Erelation_2EWF *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap$ (c_2Ebool_2E_21

Definition 25 *We define* c_2Erelation_2ERESTRICT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1$

Definition 26 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 27 *We define* c_2Erelation_2Eapprox *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 28 *We define* c_2Erelation_2Ethe__fun *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 29 *We define* c_2Erelation_2EWFREC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1M$

Definition 30 *We define* c_2Elist_2ESET__TO__LIST *to be* $\lambda A\_27a : \iota.(ap\ (ap$ (c_2Erelation_2EWFREC $(2^{A\_2}$

Definition 31 *We define* c_2Eone_2Eone *to be* $(ap$ (c_2Emin_2E_40 $ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Let $c\_2Epatricia\_2EADD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EADD\ A\_27a \in (((ty\_2Epatricia\_2Eptree\ A\_27a)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ A\_27a)})^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \tag{14}$$

Definition 32 *We define* c_2Epatricia_2EINSERT__PTREE *to be* $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1t \in (ty\_2E$

Definition 33 *We define* c_2Ecombin_2EC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Let $c\_2Elist\_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27b})^{((A\_27b^{A\_27a})^{A\_27b})}) \tag{15}$$

Definition 34 *We define* c_2Epatricia_2EPTREE__OF__NUMSET *to be* $\lambda V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2$

Let $c\_2Epatricia\_2EEmpty : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EEmpty\ A\_27a \in (ty\_2Epatricia\_2Eptree\ A\_27a) \tag{16}$$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{17}$$

Let $c\_2Epatricia\_2EPEEK : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EPEEK\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \tag{18}$$

Let $c\_2Eoption\_2EIS\_\_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_\_SOME\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{19}$$

Definition 35 *We define* c_2Epatricia_2EIN__PTREE *to be* $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1t \in (ty\_2Epatri$

Let $c\_2Epatricia\_2EIS\_\_PTREE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EIS\_\_PTREE\ A\_27a \in (2^{(ty\_2Epatricia\_2Eptree\ A\_27a)}) \tag{20}$$

Definition 36 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

Assume the following.

$$True \tag{21}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow (p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge (((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(((True\vee(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\vee True)\Leftrightarrow True)\wedge (((False\vee(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\vee False)\Leftrightarrow(p\ V0t))\wedge(((p\ V0t)\vee (p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{28}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{29}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ A\_27a)\ (c\_2Epatricia\_2EEmpty\ A\_27a))) \tag{31}$$

Assume the following.

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V0t))\Rightarrow((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V1n)\ (ap\ \mathsf{c\_2Epatricia\_2ENUMSET\_\_OF\_\_PTREE}\ V0t)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EIN\_\_PTREE}\ V1n)\ V0t)))))) \tag{32}$$

Assume the following.

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1s \in (2^{ty\_2Enum\_2Enum}).((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V0t))\Rightarrow(p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EPTREE\_\_OF\_\_NUMSET}\ V0t)\ V1s))))))\quad (33)$$

Assume the following.

$$(\forall V0s \in (2^{ty\_2Enum\_2Enum}).(p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EPTREE\_\_OF\_\_NUMSET}\ (c\_2Epatricia\_2EEmpty\ ty\_2Eone\_2Eone))\ V0s))))\quad (34)$$

Assume the following.

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Enum\_2Enum)\ (ap\ \mathsf{c\_2Epatricia\_2ENUMSET\_\_OF\_\_PTREE}\ V0t))))\quad (35)$$

Assume the following.

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1s \in (2^{ty\_2Enum\_2Enum}).(\forall V2n \in ty\_2Enum\_2Enum.(((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V0t))\wedge(p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Enum\_2Enum)\ V1s)))\Rightarrow((p\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EIN\_\_PTREE}\ V2n)\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EPTREE\_\_OF\_\_NUMSET}\ V0t)\ V1s)))\Leftrightarrow((p\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EIN\_\_PTREE}\ V2n)\ V0t))\vee(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V2n)\ V1s))))))))\quad (36)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EIN\_\_PTREE}\ V0n)\ (c\_2Epatricia\_2EEmpty\ ty\_2Eone\_2Eone)))))\quad (37)$$

Assume the following.

$$(\forall V0t1 \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1t2 \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V0t1))\wedge(p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V1t2)))\Rightarrow((V0t1 = V1t2)\Leftrightarrow(\forall V2x \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EIN\_\_PTREE}\ V2x)\ V0t1))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Epatricia\_2EIN\_\_PTREE}\ V2x)\ V1t2)))))))))\quad (38)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ V0s)\ V1t)))\Leftrightarrow((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s))\vee(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t)))))))\quad (39)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V0s)) \wedge (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ A\_27a)\ V1t)))))) \quad (40)$$

Theorem 1

$$(\forall V0t \in (ty\_2Epatricia\_2Eptree\ ty\_2Eone\_2Eone).(\forall V1s \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ (c\_2Epatricia\_2EIS\_\_PTREE\ ty\_2Eone\_2Eone)\ V0t)) \wedge (p\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EFINITE}\ ty\_2Enum\_2Enum)\ V1s))) \Rightarrow ((ap\ (ap\ \mathsf{c\_2Epatricia\_2EPTREE\_\_OF\_\_NUMSET}\ (c\_2Epatricia\_2EEmpty\ ty\_2Eone\_2Eone))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EUNION}\ ty\_2Enum\_2Enum)\ (ap\ \mathsf{c\_2Epatricia\_2ENUMSET\_\_OF\_\_PTREE}\ V0t))\ V1s)) = (ap\ (ap\ \mathsf{c\_2Epatricia\_2EPTREE\_\_OF\_\_NUMSET}\ V0t)\ V1s)))))$$