thm_2Epatricia_2Edatatype__ptree (TMVHB-bahn8rwZepDHdR8n4EwNCtR5vCRPj4)

October 26, 2020

Let $ty\_2Epatricia\_2Eptree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Epatricia\_2Eptree\ A0) \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Epatricia\_2EBranch : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EBranch\ A\_27a \in ((((ty\_2Epatricia\_2Eptree\ A\_27a)^{(ty\_2Epatricia\_2Eptree\ A\_27a)})^{(ty\_2Epatricia\_2Eptree\ A\_27a)})^{ty\_2Enum\_2Enum}) \tag{3}$$

Let $c\_2Epatricia\_2ELeaf : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2ELeaf\ A\_27a \in (((ty\_2Epatricia\_2Eptree\ A\_27a)^{A\_27a})^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Epatricia\_2EEmpty : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Epatricia\_2EEmpty\ A\_27a \in (ty\_2Epatricia\_2Eptree\ A\_27a) \tag{5}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2EDATATYPE *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2ET})$.

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Assume the following.

$$True \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((p\ (ap\ (\mathsf{c\_2Ebool\_2EDATATYPE}\ A\_27a)\ V0x)) \Leftrightarrow True)) \tag{7}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0ptree \in (((2^{(((((ty\_2Epatricia\_2Eptree\ A\_27a)^{(ty\_2Epatricia\_2Eptree\ A\_27a)})^{(ty\_2Epat}$$

$$(p\ (ap\ (\mathsf{c\_2Ebool\_2EDATATYPE}$$

$$A\_27a))\ (c\_2Epatricia\_2$$