

thm_2Epatricia__casts_2EEVERY__MAP__ORD (TMVcqD3TfoPEjjMp4rBogcsp59e4rXyjbc)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num (c_2Enum_2ESUC_REP m))$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (5)$$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EMAP \\ & A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b^{A_27a})}) \end{aligned} \quad (6)$$

Let $c_2Elist_2EEVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EEVERY\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})}) \quad (7)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (8)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (9)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Definition 13 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 14 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (11)$$

Definition 15 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic$

Definition 16 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic$

Definition 17 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $ty_2Estring_2Echar : \iota$ be given. Assume the following.

$$nonempty\ ty_2Estring_2Echar \quad (12)$$

Let $c_2Estring_2EORD : \iota$ be given. Assume the following.

$$c_2Estring_2EORD \in (ty_2Enum_2Enum^{ty_2Estring_2Echar}) \quad (13)$$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p\ V0t)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b) \\ & V0f)\ (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL\ A_27b))) \wedge (\forall V1f \in \\ & (A_27b^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist \\ & A_27a).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ (ap\ (ap\ (c_2Elist_2ECONS \\ & A_27a)\ V2h)\ V3t)) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ (ap\ V1f\ V2h)) \\ & (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ V3t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0P \in (2^{A_27a}).((p\ (ap \\ & (ap\ (c_2Elist_2EVERY\ A_27a)\ V0P)\ (c_2Elist_2ENIL\ A_27a))) \Leftrightarrow True)) \wedge \\ & (\forall V1P \in (2^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist \\ & A_27a).((p\ (ap\ (ap\ (c_2Elist_2EVERY\ A_27a)\ V1P)\ (ap\ (ap\ (c_2Elist_2ECONS \\ & A_27a)\ V2h)\ V3t))) \Leftrightarrow ((p\ (ap\ V1P\ V2h)) \wedge (p\ (ap\ (ap\ (c_2Elist_2EVERY \\ & A_27a)\ V1P)\ V3t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\ & (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ & A_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\ & c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A_27a).(p\ (ap\ V0P\ V3l)))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned}
& (\forall V0c \in ty_2Estring_2Echar.(p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Estring_2EORD V0c)) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 \\
& (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
& (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))))))) \\
& \hspace{15em} (20)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& (\forall V0l \in (ty_2Elist_2Elist ty_2Estring_2Echar).(p (ap (\\
& \hspace{2em} ap (c_2Elist_2EVERY ty_2Enum_2Enum) (ap c_2Earithmetic_2E_3E \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 \\
& (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
& (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO)))))))))) (ap (ap (c_2Elist_2EMAP ty_2Estring_2Echar \\
& \hspace{2em} ty_2Enum_2Enum) c_2Estring_2EORD) V0l)))
\end{aligned}$$