thm_2Epatricia__casts_2ETHE__PTREE__SOME__PTREE (TM-cGLAprL6A4QC9Wd8tG1AM6RKeWvGjxg54)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Epatricia\_2Eptree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Epatricia\_2Eptree\ A0) \quad (1)$$

Let $ty\_2Epatricia\_\_casts\_2Eword\_\_ptree : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epatricia\_\_casts\_2Eword\_\_ptree\ A0\ A1) \quad (2)$$

Let $c\_2Epatricia\_\_casts\_2ETHE\_\_PTREE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epatricia\_\_casts\_2ETHE\_\_PTREE\ A\_27a\ A\_27b \in ((ty\_2Epatricia\_2Eptree\ A\_27a)^{(ty\_2Epatricia\_\_casts\_2Eword\_\_ptree\ A\_27b\ A\_27a)}) \quad (3)$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (4)$$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

Definition 5 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Let $c\_2Epatricia\_\_casts\_2EWord\_\_ptree : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epatricia\_\_casts\_2EWord\_\_ptree\ A\_27a\ A\_27b \in (((ty\_2Epatricia\_\_casts\_2Eword\_\_ptree\ A\_27a\ A\_27b)^{(ty\_2Epatricia\_2Eptree\ A\_27b)})^{(ty\_2Eone\_} \tag{5}$$

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 7 *We define* c_2Epatricia__casts_2ESOME__PTREE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0t \in (ty\_2Epat$

Assume the following.

$$True \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0a \in (ty\_2Eone\_2Eone^{A\_27b}).(\forall V1t \in (ty\_2Epatricia\_2Eptree\ A\_27a).((ap\ (c\_2Epatricia\_\_casts\_2ETHE\_\_PTREE\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Epatricia\_\_casts\_2EWord\_\_ptree\ A\_27b\ A\_27a)\ V0a)\ V1t)) = V1t))) \tag{8}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0t \in (ty\_2Epatricia\_2Eptree\ A\_27a).((ap\ (c\_2Epatricia\_\_casts\_2ETHE\_\_PTREE\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Epatricia\_\_casts\_2ESOME\_\_PTREE}\ A\_27b\ A\_27a)\ V0t)) = V0t))$$