

thm_2Epatricia__casts_2Edatatype__word__ptree
(TMMwc-
dRE8eRcYvPnSK9QDbGjnaFbaCgBLYT)

October 26, 2020

Let $ty_2Epatricia_casts_2Eword_ptree : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epatricia_casts_2Eword_ptree\ A0\ A1) \quad (1)$$

Let $ty_2Epatricia_2Eptree : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Epatricia_2Eptree\ A0) \quad (2)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (3)$$

Let $c_2Epatricia_casts_2Eword_ptree : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epatricia_casts_2Eword_ptree\ A_27a\ A_27b \in (((ty_2Epatricia_casts_2Eword_ptree\ A_27a\ A_27b)^{(ty_2Epatricia_2Eptree\ A_27b)})^{(ty_2Eone_2Eone)}) \quad (4)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2E2T to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2EDATATYPE$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E2T)$.

Definition 4 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a}))\ (c_2Ebool_2E21))))$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((p\ (ap\ (c_2Ebool_2EDATATYPE\ A_27a)\ V0x)) \Leftrightarrow True)) \quad (6)$$

Theorem 1

$$\forall A_{27a}.nonempty\ A_{27a} \Rightarrow \forall A_{27b}.nonempty\ A_{27b} \Rightarrow (\forall V0word_ptree \in (2(((ty_2Epatricia_casts_2Eword_ptree\ A_{27a}\ A_{27b})^{(ty_2Epatricia_2Eptree\ A_{27b})} (ty_2Eone_2EoneA_{27a}))) (p (ap (c_2Ebool_2EDATATYPE\ 2) (ap\ V0word_ptree\ (c_2Epatricia_casts_2EWord_ptree\ A_{27a}\ A_{27b}))))))$$