

thm_2Epatricia__casts_2El2n__11 (TMRkU- jhXFHeANqtvE3t4KXChmtYVym2dHsx)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x)) \text{ of type } \iota \Rightarrow \iota$.

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40 } A \text{ (ap } P \ x))$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty ty_2Enum_2Enum} \tag{1}$$

Let `c_2Earithmetic_2EEVEN` : ι be given. Assume the following.

$$\text{c_2Earithmetic_2EEVEN} \in (2^{\text{ty_2Enum_2Enum}}) \tag{2}$$

Let `c_2Earithmetic_2EODD` : ι be given. Assume the following.

$$\text{c_2Earithmetic_2EODD} \in (2^{\text{ty_2Enum_2Enum}}) \tag{3}$$

Definition 4 We define `c_2Ebool_2ET` to be $(\text{ap (ap (c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap (ap (c_2Emin_2E_3D } (2^{A-27a})$

Definition 6 We define `c_2Ebool_2EF` to be $(\text{ap (c_2Ebool_2E_21 } 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 8 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap (ap c_2Emin_2E_3D_3D_3E } V0t) \text{ c_2Ebool_2EF } V0t))$

Definition 9 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap (c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. (\text{ap (c_2Ebool_2E_7E } V2t) \text{ c_2Ebool_2E_21 } V2t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (6)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2$

Definition 14 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Definition 15 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 17 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2E$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Definition 18 We define $c_2Enumeral_2EiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ c_2Enum_2ESUC\ (ap$

Definition 19 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Definition 20 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 21 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (10)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (11)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (12)$$

Let $c_2Elist_2EEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EEL\ A_27a \in ((A_27a^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (15)$$

Let $c_2Elist_2EEVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EEVERY\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})}) \quad (16)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (17)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (18)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (19)$$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (20)$$

Let $c_2Enumposrep_2El2n : \iota$ be given. Assume the following.

$$c_2Enumposrep_2El2n \in ((ty_2Enum_2Enum^{(ty_2Elist_2Elist\ ty_2Enum_2Enum)})^{ty_2Enum_2Enum}) \quad (21)$$

Definition 22 We define $c_2\text{Earithmetic_2EZERO}$ to be $c_2\text{Enum_2E0}$.

Definition 23 We define $c_2\text{Earithmetic_2EBIT1}$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m)$

Definition 24 We define $c_2\text{Earithmetic_2ENUMERAL}$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEXP \\ V0m) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ c_2Earithmetic_2EZERO)))) \wedge (\forall V1m \in ty_2Enum_2Enum. (\forall V2n \in \\ ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEXP V1m) (ap c_2Enum_2ESUC \\ V2n)) = (ap (ap c_2Earithmetic_2E_2A V1m) (ap (ap c_2Earithmetic_2EEXP \\ V1m) V2n)))))) \end{aligned} \quad (22)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) \\ c_2Enum_2E0) = V0m)) \quad (23)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\ ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\ (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\ V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\ V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n)))))))))) \end{aligned} \quad (24)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\ V1n) V0m)))) \quad (25)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\ V1n) V0m)))) \quad (26)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) \\ (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\ (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C V1n) V0m)) \Rightarrow (\exists V2p \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2E_2B V2p) V1n) = V0m))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Enum_2ESUC V0m)) V1n))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& c_2Enum_2E0) V0n)))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (\neg (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1n) V0m))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V0m) \\
& c_2Enum_2E0) = c_2Enum_2E0))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A (\\
& ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \\
& V0m) = V0m))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V0m) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = \\
& V0m))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge (\\
& ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
& (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
& (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
& V0m) V1n))))))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (ap (ap c_2Earithmetic_2E_2A V0m) V1n) = (ap (ap c_2Earithmetic_2E_2A \\
& V1n) V0m))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A (ap \\
& (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p) = (ap (ap c_2Earithmetic_2E_2B \\
& (ap (ap c_2Earithmetic_2E_2A V0m) V2p)) (ap (ap c_2Earithmetic_2E_2A \\
& V1n) V2p))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V2p) \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) = (ap (ap c_2Earithmetic_2E_2B \\
& (ap (ap c_2Earithmetic_2E_2A V2p) V0m)) (ap (ap c_2Earithmetic_2E_2A \\
& V2p) V1n))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2E_2B V0m) \\
& V2p) = (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) \Leftrightarrow (V0m = V1n))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C V1n) V0m)) \Rightarrow (\exists V2p \in ty_2Enum_2Enum. \\
& (V0m = (ap (ap c_2Earithmetic_2E_2B V1n) (ap (ap c_2Earithmetic_2E_2B \\
& V2p) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (V0m = V1n) \vee ((p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \vee (p (ap (ap \\
& c_2Eprim_rec_2E_3C V1n) V0m))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\
& V1n)) V0m))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\
& c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))) V0n)))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& ((p (ap (ap (ap c_2Eprim_rec_2E_3C V0m) (ap (ap c_2Earithmetic_2E_2A \\
& V0m) V1n))) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0m)) \wedge \\
& (p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V1n)))) \wedge \\
& ((p (ap (ap c_2Eprim_rec_2E_3C V0m) (ap (ap c_2Earithmetic_2E_2A \\
& V1n) V0m))) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0m)) \wedge \\
& (p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V1n)))))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1k \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C V1k) V0n)) \Rightarrow ((ap (ap c_2Earithmetic_2EMOD \\
& V1k) V0n) = V1k))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Enum_2Enum. (\forall V1y \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap (ap c_2Earithmetic_2EEXP \\
& V0x) V1y))) \Leftrightarrow ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V0x)) \vee \\
& (V1y = c_2Enum_2E0))))))
\end{aligned} \tag{49}$$

Assume the following.

$$True \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))
\end{aligned} \tag{51}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{52}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \tag{53}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\
& A_27a. (p V0t)) \Leftrightarrow (p V0t)))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge \\
& ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (56)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (57)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (58)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (59)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\
& True)) \quad (60)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (61)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t)))))) \quad (62)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (\\
& (p \ V1B) \vee (p \ V2C)) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \quad (63)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(\\
& p \ V0A) \vee (\neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A) \wedge (\neg(p \ V1B))))))))) \quad (64)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (65)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (66)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (67)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1)) \wedge (\neg(p V1t2))))))) \quad (68)$$

Assume the following.

$$2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))) \quad (69)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0l1 \in (ty_{2Elist_2Elist} A_{27a}).(\forall V1l2 \in (ty_{2Elist_2Elist} A_{27a}).(((ap (c_{2Elist_2ELENGTH} A_{27a}) V0l1) = (ap (c_{2Elist_2ELENGTH} A_{27a}) V1l2)) \wedge (\forall V2x \in ty_{2Enum_2Enum}.((p (ap (ap c_{2Eprim_rec_2E_3C} V2x) (ap (c_{2Elist_2ELENGTH} A_{27a}) V0l1))) \Rightarrow ((ap (ap (c_{2Elist_2EEL} A_{27a}) V2x) V0l1) = (ap (ap (c_{2Elist_2EEL} A_{27a}) V2x) V1l2)))))) \Rightarrow (V0l1 = V1l2)))))) \quad (70)$$

Assume the following.

$$(\forall V0P \in (2^{ty_{2Enum_2Enum}}).(((p (ap V0P c_{2Enum_2E0})) \wedge (\forall V1n \in ty_{2Enum_2Enum}.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c_{2Enum_2ESUC} V1n)))))) \Rightarrow (\forall V2n \in ty_{2Enum_2Enum}.(p (ap V0P V2n)))))) \quad (71)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m)))))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0b \in ty_2Enum_2Enum. ((ap (ap c_2Enumposrep_2El2n V0b) \\
& (c_2Elist_2ENIL ty_2Enum_2Enum)) = c_2Enum_2E0)) \wedge (\forall V1b \in \\
& ty_2Enum_2Enum. (\forall V2h \in ty_2Enum_2Enum. (\forall V3t \in (\\
& ty_2Elist_2Elist ty_2Enum_2Enum). ((ap (ap c_2Enumposrep_2El2n \\
& V1b) (ap (ap (c_2Elist_2ECONS ty_2Enum_2Enum) V2h) V3t)) = (ap (\\
& ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2EMOD V2h) V1b)) \\
& (ap (ap c_2Earithmetic_2E_2A V1b) (ap (ap c_2Enumposrep_2El2n \\
& V1b) V3t)))))))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty_2Enum_2Enum. (\forall V1l \in (ty_2Elist_2Elist \\
& ty_2Enum_2Enum). (\forall V2x \in ty_2Enum_2Enum. (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \\
& V0b)) \wedge ((p (ap (ap (c_2Elist_2EVERY ty_2Enum_2Enum) (ap c_2Earithmetic_2E_3E \\
& V0b)) V1l)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C V2x) (ap (c_2Elist_2ELENGTH \\
& ty_2Enum_2Enum) V1l)))))) \Rightarrow ((ap (ap c_2Earithmetic_2EMOD (ap (\\
& ap c_2Earithmetic_2EDIV (ap (ap c_2Enumposrep_2El2n V0b) V1l)) \\
& (ap (ap c_2Earithmetic_2EEXP V0b) V2x))) V0b) = (ap (ap (c_2Elist_2EEL \\
& ty_2Enum_2Enum) V2x) V1l))))))
\end{aligned} \tag{76}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty_2Enum_2Enum. (\forall V1l1 \in (ty_2Elist_2Elist \\
& \quad ty_2Enum_2Enum). (\forall V2l2 \in (ty_2Elist_2Elist \ ty_2Enum_2Enum). \\
& \quad ((ap (ap c_2Enumposrep_2El2n V0b) (ap (ap (c_2Elist_2EAPPEND \ ty_2Enum_2Enum) \\
& \quad \quad V1l1) \ V2l2))) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Enumposrep_2El2n \\
& \quad \quad V0b) \ V1l1)) (ap (ap c_2Earithmetic_2E_2A (ap (ap c_2Earithmetic_2EEXP \\
& \quad \quad V0b) (ap (c_2Elist_2ELENGTH \ ty_2Enum_2Enum) \ V1l1)))) (ap (ap c_2Enumposrep_2El2n \\
& \quad \quad \quad V0b) \ V2l2))))))
\end{aligned} \tag{77}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty_2Enum_2Enum. (\forall V1l \in (ty_2Elist_2Elist \\
& \quad ty_2Enum_2Enum). ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad \quad (ap c_2Earithmetic_2EBIT1 \ c_2Earithmetic_2EZERO))) \ V0b)) \Rightarrow (\\
& \quad \quad p (ap (ap c_2Eprim_rec_2E_3C (ap (ap c_2Enumposrep_2El2n \ V0b) \\
& \quad \quad V1l)) (ap (ap c_2Earithmetic_2EEXP \ V0b) (ap (c_2Elist_2ELENGTH \\
& \quad \quad \quad ty_2Enum_2Enum) \ V1l))))))
\end{aligned} \tag{78}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg (p (ap (ap c_2Eprim_rec_2E_3C \ V0n) \ c_2Enum_2E0)))) \tag{79}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \tag{80}$$

Assume the following.

$$(\forall V0A \in 2. ((p \ V0A) \Rightarrow ((\neg(p \ V0A)) \Rightarrow False))) \tag{81}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p \ V0A) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow \\
((p \ V0A) \Rightarrow False) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \tag{82}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p \ V0A)) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow \\
((p \ V0A) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \tag{83}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p \ V0A)) \Rightarrow False) \Rightarrow (((p \ V0A) \Rightarrow False) \Rightarrow False))) \tag{84}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& \quad (p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg \\
& \quad p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee \\
& \quad \quad ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \wedge (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (\neg(p \vee V1q)) \vee \neg(p \vee V2r))) \wedge (((p \vee V1q) \vee \\
& (\neg(p \vee V0p))) \wedge ((p \vee V2r) \vee \neg(p \vee V0p))))))))) \\
& \tag{86}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \vee (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee \neg(p \vee V1q)) \wedge ((p \vee V0p) \vee \neg(p \vee V2r))) \wedge \\
& ((p \vee V1q) \vee ((p \vee V2r) \vee \neg(p \vee V0p))))))))) \\
& \tag{87}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee V0p) \Leftrightarrow (\\
& (p \vee V1q) \Rightarrow (p \vee V2r))) \Leftrightarrow (((p \vee V0p) \vee (p \vee V1q)) \wedge (((p \vee V0p) \vee \neg(p \vee V2r))) \wedge (\\
& \neg(p \vee V1q) \vee ((p \vee V2r) \vee \neg(p \vee V0p))))))))) \\
& \tag{88}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee V0p) \Leftrightarrow \neg(p \vee V1q)) \Leftrightarrow (((p \vee V0p) \vee \\
& (p \vee V1q)) \wedge (\neg(p \vee V1q) \vee \neg(p \vee V0p)))))) \\
& \tag{89}
\end{aligned}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee V0p) \Rightarrow (p \vee V1q)) \Rightarrow (p \vee V0p))) \tag{90}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee V0p) \Rightarrow (p \vee V1q)) \Rightarrow \neg(p \vee V1q))) \tag{91}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee V0p) \vee (p \vee V1q)) \Rightarrow \neg(p \vee V0p))) \tag{92}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \vee V0p) \vee (p \vee V1q)) \Rightarrow \neg(p \vee V1q))) \tag{93}$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p \vee V0p)) \Rightarrow (p \vee V0p))) \tag{94}$$

Theorem 1

$$\begin{aligned} & (\forall V0b \in ty_2Enum_2Enum. (\forall V1l1 \in (ty_2Elist_2Elist \\ & ty_2Enum_2Enum). (\forall V2l2 \in (ty_2Elist_2Elist ty_2Enum_2Enum). \\ & (((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0b)) \wedge (\\ & (p (ap (ap (c_2Elist_2EVERY ty_2Enum_2Enum) (ap c_2Earithmetic_2E_3E \\ & V0b)) V1l1)) \wedge (p (ap (ap (c_2Elist_2EVERY ty_2Enum_2Enum) (ap \\ & c_2Earithmetic_2E_3E V0b)) V2l2)))) \Rightarrow (((ap (ap c_2Enumposrep_2El2n \\ & V0b) (ap (ap (c_2Elist_2EAPPEND ty_2Enum_2Enum) V1l1) (ap (ap (\\ & c_2Elist_2ECONS ty_2Enum_2Enum) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) (c_2Elist_2ENIL \\ & ty_2Enum_2Enum)))) = (ap (ap c_2Enumposrep_2El2n V0b) (ap (ap (\\ & c_2Elist_2EAPPEND ty_2Enum_2Enum) V2l2) (ap (ap (c_2Elist_2ECONS \\ & ty_2Enum_2Enum) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO))) (c_2Elist_2ENIL ty_2Enum_2Enum)))))) \Leftrightarrow \\ & (V1l1 = V2l2)))))) \end{aligned}$$