thm_2Epatricia__casts_2Enum__to__string__string__to__num (TMb-wnP7DahcHByJKhpPcC6FFUNH5JRTxTNw)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t)).$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Estring\_2Echar : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Estring\_2Echar \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Estring\_2ECHR : \iota$ be given. Assume the following.

$$c\_2Estring\_2ECHR \in (ty\_2Estring\_2Echar^{ty\_2Enum\_2Enum}) \tag{3}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{4}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{5}$$

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Definition 8 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 9 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (6)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (7)$$

Definition 10 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(8)$$

Definition 11 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 12 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 13 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \qquad (9)$$

Let $c\_2Enumposrep\_2En2l : \iota$ be given. Assume the following.

$$c\_2Enumposrep\_2En2l \in (((ty\_2Elist\_2Elist\ ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(10)$$

Let $c\_2Elist\_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EMAP$$
$$A\_27a\ A\_27b \in (((ty\_2Elist\_2Elist\ A\_27b)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(A\_27b^{A\_27a})})$$
$$(11)$$

Let $c\_2Elist\_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EREVERSE\ A\_27a \in ((ty\_2Elist\_2Elist$$
$$A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})$$
$$(12)$$

Definition 14 *We define* c_2EASCIInumbers_2En2s *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1f \in (ty\_2Estring$

Let $c\_2Epatricia\_\_casts\_2ESKIP1 : \iota$ be given. Assume the following.

$$c\_2Epatricia\_\_casts\_2ESKIP1 \in ((ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar)^{(ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar}}$$
$$(13)$$

Definition 15 *We define* c_2Epatricia__casts_2Enum__to__string *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ c\_2Ep$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist \\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})$$

$$(14)$$

Let $c\_2Estring\_2EORD : \iota$ be given. Assume the following.

$$c\_2Estring\_2EORD \in (ty\_2Enum\_2Enum^{ty\_2Estring\_2Echar}) \qquad (15)$$

Let $c\_2Enumposrep\_2El2n : \iota$ be given. Assume the following.

$$c\_2Enumposrep\_2El2n \in ((ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ ty\_2Enum\_2Enum)})^{ty\_2Enum\_2Enum})$$

$$(16)$$

Definition 16 *We define* c_2EASCIInumbers_2Es2n *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1f \in (ty\_2Enum\_2$

Definition 17 *We define* c_2Epatricia__casts_2Estring__to__num *to be* $\lambda V0s \in (ty\_2Elist\_2Elist\ ty\_2Estring$

Definition 18 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c_2Ebool_2ET)$.

Definition 19 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod \\ A0\ A1)$$

$$(17)$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})})$$

$$(18)$$

Definition 20 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})})$$

$$(19)$$

Definition 21 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

Assume the following.

$$True \qquad (20)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\ True)) \qquad (21)$$

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Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\ p\ V0t)))))))) \tag{23}$$

Assume the following.

$$(\forall V0s \in (ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar).(\forall V1t \in \\ (ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar).(((ap\ \mathsf{c\_2Epatricia\_\_casts\_2Estring\_\_to\_\_num} \\ V0s) = (ap\ \mathsf{c\_2Epatricia\_\_casts\_2Estring\_\_to\_\_num}\ V1t)) \Leftrightarrow (V0s = \\ V1t)))) \tag{24}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum) \\ V0n)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ (ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar) \\ ty\_2Enum\_2Enum)\ \mathsf{c\_2Epatricia\_\_casts\_2Estring\_\_to\_\_num})\ (\mathsf{c\_2Epred\_\_set\_2EUNIV} \\ (ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar))))) \Rightarrow ((ap\ \mathsf{c\_2Epatricia\_\_casts\_2Estring\_\_to\_\_num} \\ (ap\ \mathsf{c\_2Epatricia\_\_casts\_2Enum\_\_to\_\_string}\ V0n)) = V0n))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN} \\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ A\_27a)))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\\ \forall V0x \in A\_27a.(\forall V1s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN} \\ A\_27a)\ V0x)\ V1s)) \Rightarrow (\forall V2f \in (A\_27b^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN} \\ A\_27b)\ (ap\ V2f\ V0x))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b) \\ V2f)\ V1s))))))) \tag{27}$$

Theorem 1

$$(\forall V0s \in (ty\_2Elist\_2Elist\ ty\_2Estring\_2Echar).((ap\ \mathsf{c\_2Epatricia\_\_casts\_2Enum\_\_to\_\_string} \\ (ap\ \mathsf{c\_2Epatricia\_\_casts\_2Estring\_\_to\_\_num}\ V0s)) = V0s))$$