

# thm\_2EpatternMatches\_2EFST\_\_STRONGEST\_\_REDUNDANT\_\_P (TMQSCp7wHq77VKJsmiA7Md9i3RcTPGq7vt7)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 6** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2EEVERY : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EEVERY A\_27a \in ((2^{(ty\_2Elist\_2Elist A\_27a)})^{(2^{A\_27a})}) \quad (2)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (3)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EFST A\_27a A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \quad (4)$$

Let  $c\_2Elist\_2ESNOC : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ESNOC\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})_{A\_27a})_{A\_27a} \quad (5)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (6)$$

**Definition 8** We define  $c\_2Emin\_2E40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p\ (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 9** We define  $c\_2Eone\_2Eone$  to be  $(ap\ (c\_2Emin\_2E40\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2Eone$

**Definition 10** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (7)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \quad (8)$$

**Definition 11** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (9)$$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (10)$$

**Definition 12** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_ABS\ A\_27a)\ ($

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})_{A\_27a})_{A\_27a} \quad (11)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (12)$$

**Definition 13** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2E\_2C) A\_27a A\_27b)$ .  
Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (13)$$

Let  $c\_2EpatternMatches\_2ESTRONGEST\_REDUNDANT\_ROWS\_INFO\_AUX : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2EpatternMatches\_2ESTRONGEST\_REDUNDANT\_ROWS\_INFO\_AUX\ A\_27a\ A\_27b \in (((((ty\_2Epair\_2Eprod\ 2\ (ty\_2Elist\_2Elist\ 2))^{(ty\_2Elist\_2Elist\ 2)})^2)^{(ty\_2Elist\_2Elist\ ((ty\_2Elist\_2Elist\ 2))})}))) \quad (14)$$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee \neg(p\ V0t))) \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (20)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (21)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\
& p V0t))))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow ((\forall V0P \in (2^{A.27a}).((p (ap \\
& (ap (c.2Elist.2EVERY A.27a) V0P) (c.2Elist.2ENIL A.27a))) \Leftrightarrow True)) \wedge \\
& (\forall V1P \in (2^{A.27a}).(\forall V2h \in A.27a.(\forall V3t \in (ty.2Elist.2Elist \\
& A.27a).((p (ap (ap (c.2Elist.2EVERY A.27a) V1P) (ap (ap (c.2Elist.2ECONS \\
& A.27a) V2h) V3t))) \Leftrightarrow ((p (ap V1P V2h)) \wedge (p (ap (ap (c.2Elist.2EVERY \\
& A.27a) V1P) V3t))))))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{(ty.2Elist.2Elist A.27a)}). \\
& (((p (ap V0P (c.2Elist.2ENIL A.27a))) \wedge (\forall V1t \in (ty.2Elist.2Elist \\
& A.27a).((p (ap V0P V1t)) \Rightarrow (\forall V2h \in A.27a.(p (ap V0P (ap (ap ( \\
& c.2Elist.2ECONS A.27a) V2h) V1t)))))) \Rightarrow (\forall V3l \in (ty.2Elist.2Elist \\
& A.27a).(p (ap V0P V3l))))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\
& \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap (c.2Epair.2EFST A.27a \\
& A.27b) (ap (ap (c.2Epair.2E.2C A.27a A.27b) V0x) V1y)) = V0x)))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ( \\
& (\forall V0v \in A.27a.(\forall V1p \in 2.(\forall V2infos \in (ty.2Elist.2Elist \\
& 2).((ap (ap (ap (ap (c.2EpatternMatches.2ESTRONGEST\_REDUNDANT\_1 \\
& A.27a A.27b) V0v) (c.2Elist.2ENIL ((ty.2Eoption.2Eoption A.27b \\
& V1p) V2infos) = (ap (ap (c.2Epair.2E.2C 2 (ty.2Elist.2Elist \\
& V1p) V2infos)))))) \wedge (\forall V3v \in A.27a.(\forall V4r \in ((ty.2Eoption.2Eop \\
& A.27b)^{A.27a}).(\forall V5rows \in (ty.2Elist.2Elist ((ty.2Eoption.2Eop \\
& A.27b)^{A.27a}).(\forall V6p \in 2.(\forall V7infos \in (ty.2Elist.2Elist \\
& 2).((ap (ap (ap (ap (c.2EpatternMatches.2ESTRONGEST\_REDUNDANT\_1 \\
& A.27a A.27b) V3v) (ap (ap (c.2Elist.2ECONS ((ty.2Eoption.2E \\
& A.27b)^{A.27a})) V4r) V5rows)) V6p) V7infos) = (ap (ap (ap (ap (c.2EpatternMatches.2ESTRONGES \\
& A.27a A.27b) V3v) V5rows) (ap (ap c.2Ebool.2E.2F.5C V6p) (a \\
& (c.2Emin.2E.3D (ty.2Eoption.2Eoption A.27b)) (ap V4r V3v)) (c.2Eopt \\
& A.27b)))) (ap (ap (c.2Elist.2ESNOC 2) (ap (ap c.2Emin.2E.3D. \\
& V6p) (ap (ap (c.2Emin.2E.3D (ty.2Eoption.2Eoption A.27b)) \\
& V4r V3v)) (c.2Eoption.2ENONE A.27b)))) V7infos))))))
\end{aligned} \tag{27}$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0v \in A\_27a. (\forall V1rows \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption \\ & A\_27b)^{A\_27a})). (\forall V2p \in 2. (\forall V3infos \in (ty\_2Elist\_2Elist \\ & 2). ((p\ (ap\ (c\_2Epair\_2EFST\ 2\ (ty\_2Elist\_2Elist\ 2))\ (ap\ (ap\ ( \\ & ap\ (ap\ (c\_2EpatternMatches\_2ESTRONGEST\_REDUNDANT\_ROWS\_INFO\_AUX \\ & A\_27a\ A\_27b)\ V0v)\ V1rows)\ V2p)\ V3infos))) \Leftrightarrow ((p\ V2p) \wedge (p\ (ap\ (ap\ (c\_2Elist\_2EVERY \\ & ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}))\ (\lambda V4r \in ((ty\_2Eoption\_2Eoption \\ & A\_27b)^{A\_27a}). (ap\ (ap\ (c\_2Emin\_2E\_3D\ (ty\_2Eoption\_2Eoption\ A\_27b)) \\ & (ap\ V4r\ V0v))\ (c\_2Eoption\_2ENONE\ A\_27b))))\ V1rows)))))) \end{aligned}$$