thm_2EpatternMatches_2EGUARDS__ELIM__THM (TMTHWtVZkTV5VZpc3rmycqmqb9XYcMX5qMM)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \tag{3}$$

Definition 10 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0e\in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (4)$$

Let $c\_2Eoption\_2Eoption\_\_ABS:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a\in((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (5)$$

Definition 11 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a:\iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 12 *We define* c_2EpatternMatches_2EPMATCH__ROW__COND *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0pat\in(A\_27b^{A\_27a}).\lambda V1guard\in(2^{A\_27a}).\lambda V2inp\in A\_27b.\lambda V3v\in A\_27a.(ap\ (ap$

Definition 13 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0e\in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 14 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a:\iota.\lambda V0x\in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 15 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a:\iota.(\lambda V0P\in(2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 16 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a:\iota.(\lambda V0t\in 2.(\lambda V1t1\in A\_27a.(\lambda V2t2\in A\_27a.($

Definition 17 *We define* c_2Eoption_2Esome *to be* $\lambda A_27a:\iota.\lambda V0P\in(2^{A_27a}).(ap\ (ap\ (ap\ ($c_2Ebool_2EC

Let $c\_2Eoption\_2EOPTION\_\_MAP:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b\in(((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \quad (6)$$

Definition 18 *We define* c_2EpatternMatches_2EPMATCH__ROW *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda A\_27c:\iota.\lambda V$

Let $ty\_2Elist\_2Elist:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (7)$$

Let $c\_2Elist\_2ECONS:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Elist\_2ECONS\ A\_27a\in(((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (8)$$

Let $c\_2Ebool\_2EARB:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ebool\_2EARB\ A\_27a\in A\_27a \quad (9)$$

Definition 19 *We define* c_2EpatternMatches_2EPMATCH__INCOMPLETE *to be* $\lambda A\_27a:\iota.(c\_2Ebool\_2EARB\ A\_27a)$.

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (10)$$

Let $c\_2Eoption\_2EIS\_\_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_\_SOME\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (11)$$

Let $c\_2Elist\_2ELIST\_\_TO\_\_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (12)$$

Definition 20 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (13)$$

Let $c\_2EpatternMatches\_2EPMATCH : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))})^{A\_27b}) \quad (14)$$

Definition 21 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{21}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Emin\_2E\_40}\ A\_27a)\ (\lambda V1y \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ V1y)\ V0x))) = V0x)) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \tag{29}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \tag{30}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{31}$$

Assume the following.

$$(\forall V0P \in 2.(\forall V1P\_27 \in 2.(\forall V2Q \in 2.(\forall V3Q\_27 \in 2.((((p\ V2Q) \Rightarrow ((p\ V0P) \Leftrightarrow (p\ V1P\_27))) \wedge ((p\ V1P\_27) \Rightarrow ((p\ V2Q) \Leftrightarrow (p\ V3Q\_27)))) \Rightarrow (((p\ V0P) \wedge (p\ V2Q)) \Leftrightarrow ((p\ V1P\_27) \wedge (p\ V3Q\_27)))))))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.(\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27))))))))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in A\_27a.(\exists V1x \in A\_27a.(V1x = V0a))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Eoption\_2Eoption\ A\_27a).((\neg(p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ V0x))) \Leftrightarrow (V0x = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0v \in A\_27b. (\forall V1p \in (A\_27b^{A\_27d}).(\forall V2g \in (2^{A\_27d}).(\forall V3r \in (A\_27c^{A\_27d}).(\forall V4rs \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})).(((ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27b)\ V0v)\ (c\_2Elist\_2ENIL\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))) = (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_INCOMPLETE}\ A\_27a)) \wedge ((ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27c\ A\_27b)\ V0v)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b}))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW}\ A\_27c\ A\_27d\ A\_27b)\ V1p)\ V2g)\ V3r))\ V4rs)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27c)\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ A\_27d)\ (\lambda V5x \in A\_27d.(ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW\_\_COND}\ A\_27d\ A\_27b)\ V1p)\ V2g)\ V0v)\ V5x))))\ (ap\ V3r\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A\_27d)\ (\lambda V6x \in A\_27d. (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW\_\_COND}\ A\_27d\ A\_27b)\ V1p)\ V2g)\ V0v)\ V6x)))))\ (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27c\ A\_27b)\ V0v)\ V4rs))))))))) \quad (37)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0v \in A\_27a.(\forall V1rows1 \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a})).(\forall V2rows2 \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a})).((ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27b\ A\_27a)\ V0v)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}))\ V1rows1)\ V2rows2)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27b)\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}))\ (\lambda V3r \in ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}).(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}))\ V3r)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}))\ V1rows1)))\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27b)\ (ap\ V3r\ V0v))))))\ (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27b\ A\_27a)\ V0v)\ V1rows1))\ (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27b\ A\_27a)\ V0v)\ V2rows2)))))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{42}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{43}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{44}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{45}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{46}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{47}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{48}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{49}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{50}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{51}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{52}$$

Theorem 1

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\begin{gathered}
\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
nonempty\ A\_27c \Rightarrow (\forall V0v \in A\_27a.(\forall V1rs1 \in (ty\_2Elist\_2Elist \\
((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a})).(\forall V2rs2 \in (ty\_2Elist\_2Elist \\
((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a})).(\forall V3p \in (A\_27a^{A\_27c}). \\
(\forall V4g \in (2^{A\_27c}).(\forall V5r \in (A\_27b^{A\_27c}).((\forall V6x1 \in \\
A\_27c.(\forall V7x2 \in A\_27c.(((ap\ V3p\ V6x1) = (ap\ V3p\ V7x2)) \Rightarrow (V6x1 = \\
V7x2)))) \Rightarrow ((ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27b\ A\_27a) \\
V0v)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a})) \\
V1rs1)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a})) \\
(ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW}\ A\_27b\ A\_27c\ A\_27a) \\
V3p)\ V4g)\ V5r))\ V2rs2))) = (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH \\
A\_27b\ A\_27a)\ V0v)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ ((ty\_2Eoption\_2Eoption \\
A\_27b)^{A\_27a}))\ V1rs1)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption \\
A\_27b)^{A\_27a}))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW} \\
A\_27b\ A\_27c\ A\_27a)\ V3p)\ (\lambda V8x \in A\_27c.\mathsf{c\_2Ebool\_2ET}))\ (\lambda V9x \in \\
A\_27c.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27b)\ (ap\ V4g\ V9x))\ (ap\ V5r\ V9x)) \\
(ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27b\ A\_27a)\ (ap\ V3p\ V9x)) \\
V2rs2)))))\ V2rs2)))))))))))
\end{gathered}
$$