

thm\_2EpatternMatches\_2EIS\_\_REDUNDANT\_\_ROWS\_\_INFO\_\_CO  
(TMFPSiWdsuKvoc41zEehiR.Jom78hSkE5JJ1)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_21$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (1)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ENIL A\_27a \in (ty\_2Elist\_2Elist A\_27a) \quad (2)$$

Let  $c\_2Elist\_2EHD : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EHD A\_27a \in (A\_27a^{(ty\_2Elist\_2Elist A\_27a)}) \quad (3)$$

**Definition 5** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ ).

**Definition 6** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a P))$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty ty\_2Eone\_2Eone \quad (4)$$

**Definition 7** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Esum\_2Esum A0 A1) \quad (5)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Esum\_2EABS\_sum A\_27a A\_27b \in ((ty\_2Esum\_2Esum A\_27a A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \quad (6)$$

**Definition 9** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap (c\_2Esum\_2EABS\_sum$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Eoption\_2Eoption A0) \quad (7)$$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS A\_27a \in ((ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Esum\_2Esum A\_27a ty\_2Eone\_2Eone)}) \quad (8)$$

**Definition 10** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap (c\_2Eoption\_2Eoption\_ABS$

**Definition 11** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21 2)$

**Definition 12** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (9)$$

**Definition 13** We define  $c\_2Eone\_2Eone$  to be  $(ap (c\_2Emin\_2E\_40 ty\_2Eone\_2Eone) (\lambda V0x \in ty\_2Eone\_2Eone 2$

**Definition 14** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap (c\_2Esum\_2EABS\_sum$

**Definition 15** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota.(ap (c\_2Eoption\_2Eoption\_ABS A\_27a) (c\_2Eone\_2Eone$

Let  $c\_2Elist\_2EEVERY : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EEVERY A\_27a \in ((2^{(ty\_2Elist\_2Elist A\_27a)})^{(2^{A\_27a})}) \quad (10)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (11)$$

Let  $c\_2Elist\_2EEL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEL\ A\_27a \in ((A\_27a^{(ty\_2Elist\_2Elist\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (12)$$

Let  $c\_2Eoption\_2EIS\_SOME : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_SOME\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (13)$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (14)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (15)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (16)$$

**Definition 16** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

**Definition 17** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (17)$$

**Definition 18** We define  $c\_2EpatternMatches\_2EPMATCH\_ROW\_REDUNDANT$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0v \in A\_27a.\lambda V1rs \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}))$

**Definition 19** We define  $c\_2EpatternMatches\_2EIS\_REDUNDANT\_ROWS\_INFO$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0v \in A\_27a.\lambda V1rows \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (18)$$

**Definition 20** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((V0m = c\_2Enum\_2E0) \vee (\exists V1n \in ty\_2Enum\_2Enum.(V0m = (ap\ c\_2Enum\_2ESUC\ V1n)))))) \quad (19)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ (ap\ c\_2Enum\_2ESUC\ V0m))\ (ap\ c\_2Enum\_2ESUC\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V1n)))) \quad (20)$$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p V0t1) \Rightarrow (p V1t2)) \Rightarrow ((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t))))) \quad (26)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (28)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (30)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (31)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (32)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0a \in A_{.27a}.(\exists V1x \in A_{.27a}.(V1x = V0a))) \quad (33)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0h \in A_{.27a}.(\forall V1t \in (ty\_2Elist\_2Elist A_{.27a}).((ap (c\_2Elist\_2EHD A_{.27a}) (ap (ap (c\_2Elist\_2ECONS A_{.27a}) V0h) V1t)) = V0h))) \quad (34)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (((ap (c\_2Elist\_2ELENGTH A_{.27a}) (c\_2Elist\_2ENIL A_{.27a})) = c\_2Enum\_2E0) \wedge (\forall V0h \in A_{.27a}.(\forall V1t \in (ty\_2Elist\_2Elist A_{.27a}).((ap (c\_2Elist\_2ELENGTH A_{.27a}) (ap (ap (c\_2Elist\_2ECONS A_{.27a}) V0h) V1t)) = (ap c\_2Enum\_2ESUC (ap (c\_2Elist\_2ELENGTH A_{.27a}) V1t))))))) \quad (35)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow ((\forall V0P \in (2^{A_{.27a}}).((p (ap (ap (c\_2Elist\_2EVERY A_{.27a}) V0P) (c\_2Elist\_2ENIL A_{.27a}))) \Leftrightarrow True)) \wedge (\forall V1P \in (2^{A_{.27a}}).(\forall V2h \in A_{.27a}.(\forall V3t \in (ty\_2Elist\_2Elist A_{.27a}).((p (ap (ap (ap (c\_2Elist\_2EVERY A_{.27a}) V1P) (ap (ap (c\_2Elist\_2ECONS A_{.27a}) V2h) V3t))) \Leftrightarrow ((p (ap V1P V2h)) \wedge (p (ap (ap (c\_2Elist\_2EVERY A_{.27a}) V1P) V3t)))))))))) \quad (36)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1l \in A_{.27b}.(\forall V2ls \in (ty\_2Elist\_2Elist A_{.27b}).(((ap (c\_2Elist\_2EEL A_{.27a}) c\_2Enum\_2E0) = (c\_2Elist\_2EHD A_{.27a})) \wedge ((ap (ap (c\_2Elist\_2EEL A_{.27b}) (ap c\_2Enum\_2ESUC V0n)) (ap (ap (c\_2Elist\_2ECONS A_{.27b}) V1l) V2ls)) = (ap (ap (c\_2Elist\_2EEL A_{.27b}) V0n) V2ls)))))) \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption \\ A.27a).((V0opt = (c\_2Eoption\_2ENONE\ A.27a)) \vee (\exists V1x \in A.27a. \\ (V0opt = (ap\ (c\_2Eoption\_2ESOME\ A.27a)\ V1x)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\neg((c\_2Eoption\_2ENONE \\ A.27a) = (ap\ (c\_2Eoption\_2ESOME\ A.27a)\ V0x)))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ \forall V0v \in A.27a.(\forall V1r \in ((ty\_2Eoption\_2Eoption\ A.27b)^{A.27a}). \\ (\forall V2rs \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A.27b)^{A.27a})). \\ ((p\ (ap\ (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\_ROW\_REDUNDANT \\ A.27a\ A.27b)\ V0v)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption \\ A.27b)^{A.27a}))\ V1r)\ V2rs))\ c\_2Enum\_2E0)) \Leftrightarrow ((ap\ V1r\ V0v) = (c\_2Eoption\_2ENONE \\ A.27b)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ \forall V0v \in A.27a.(\forall V1r \in ((ty\_2Eoption\_2Eoption\ A.27b)^{A.27a}). \\ (\forall V2rs \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A.27b)^{A.27a})). \\ (\forall V3i \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\_ROW\_REDUNDANT \\ A.27a\ A.27b)\ V0v)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption \\ A.27b)^{A.27a}))\ V1r)\ V2rs))\ (ap\ c\_2Enum\_2ESUC\ V3i))) \Leftrightarrow (((\neg((ap\ V1r \\ V0v) = (c\_2Eoption\_2ENONE\ A.27b))) \wedge (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\ V3i)\ (ap\ (c\_2Elist\_2ELENGTH\ ((ty\_2Eoption\_2Eoption\ A.27b)^{A.27a})) \\ V2rs)))) \vee (p\ (ap\ (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\_ROW\_REDUNDANT \\ A.27a\ A.27b)\ V0v)\ V2rs)\ V3i)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ ((ap\ c\_2Enum\_2ESUC\ V0m) = (ap\ c\_2Enum\_2ESUC\ V1n)) \Leftrightarrow (V0m = V1n)))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} (\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0) \\ (ap\ c\_2Enum\_2ESUC\ V0n)))) \end{aligned} \quad (43)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0v \in A\_27a. (\forall V1row \in ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}). \\ & \quad (\forall V2rows \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a})). \\ & \quad (\forall V3c \in 2. (\forall V4i \in 2. (\forall V5infos\_27 \in (ty\_2Elist\_2Elist \\ & \quad 2). ((p\ (ap\ (ap\ (ap\ (ap\ (c\_2EpatternMatches\_2EIS\_REDUNDANT\_ROWS\_INFO \\ & \quad A\_27a\ A\_27b)\ V0v)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption \\ & \quad A\_27b)^{A\_27a}))\ V1row)\ V2rows))\ V3c)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ 2) \\ & \quad V4i)\ V5infos\_27))) \Leftrightarrow (((ap\ (c\_2Elist\_2ELENGTH\ ((ty\_2Eoption\_2Eoption \\ & \quad A\_27b)^{A\_27a}))\ V2rows) = (ap\ (c\_2Elist\_2ELENGTH\ 2)\ V5infos\_27)) \wedge \\ & \quad (((p\ V4i) \Rightarrow ((ap\ V1row\ V0v) = (c\_2Eoption\_2ENONE\ A\_27b))) \wedge ((ap \\ & \quad V1row\ V0v) = (c\_2Eoption\_2ENONE\ A\_27b))) \Rightarrow (p\ (ap\ (ap\ (ap\ (ap\ (c\_2EpatternMatches\_2EIS\_REDUNDA \\ & \quad A\_27a\ A\_27b)\ V0v)\ V2rows)\ V3c)\ V5infos\_27))))))))) \end{aligned}$$