

thm_2EpatternMatches_2EPMATCH__APPEND__SEM (TMYuuV62DGLkwnJAN6LmJ1Gu9kRu1vuW4AG)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define $c_2Ebool_2E_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow q Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ebool_2E_2ECOND$ to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Emin_2E_40 (2^{A_27a}))$

Definition 9 We define $c_2Ebool_2E_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2E_2EF$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (1)$$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EAPPEND A_27a \in (((ty_2Elist_2Elist A_27a)(ty_2Elist_2Elist A_27a))(ty_2Elist_2Elist A_27a)) \quad (2)$$

Let $c_2Elist_2EELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EELIST_TO_SET A_27a \in ((2^{A_27a})(ty_2Elist_2Elist A_27a)) \quad (3)$$

Definition 10 We define c_Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 11 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ V0P))))$

Definition 12 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (V1t2\ V0t1))))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (4)$$

Definition 13 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (5)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (6)$$

Definition 14 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (7)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (8)$$

Definition 15 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (ty_2Eone_2Eone))$

Definition 16 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 17 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Let $c_2Eoption_2EIS_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Eoption_2EIS_SOME\ A_27a \in (2^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (9)$$

Definition 18 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Definition 19 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c)^{A_27b})^{A_27a})$

Definition 20 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ A_27a)^{A_27a})\ A_27a)$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2Eoption_CASE\ A_27a\ A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (10)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (11)$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (12)$$

Definition 21 We define $c_2EpatternMatches_2EPMATCH_INCOMPLETE$ to be $\lambda A_27a : \iota.(c_2Ebool_2EARB\ A_27a)$.

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (13)$$

Let $c_2EpatternMatches_2EPMATCH : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2EpatternMatches_2EPMATCH\ A_27a\ A_27b \in ((A_27a^{(ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption\ A_27a)^{A_27b}))})^{A_27b}) \quad (14)$$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (21)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ (2^{A_27a}). ((\exists V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ ((\exists V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \vee (\exists V4x \in A_27a. (p\ (\\ ap\ V1Q\ V4x))))))) \end{aligned} \quad (23)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \vee \\ (p\ V2C)) \wedge (p\ V0A)) \Leftrightarrow (((p\ V1B) \wedge (p\ V0A)) \vee ((p\ V2C) \wedge (p\ V0A)))))) \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (25)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\ V5y_27)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in \\ A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (\\ ap\ V0P\ V1a)))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2El\ A_27a)\ V0x) = V0x)) \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & ((\forall V0l \in (ty_2Elist_2Elist\ A_27a). ((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ (c_2Elist_2ENIL\ A_27a))\ V0l) = V0l)) \wedge (\forall V1l1 \in (ty_2Elist_2Elist\ A_27a). (\forall V2l2 \in \\ & (ty_2Elist_2Elist\ A_27a). (\forall V3h \in A_27a. ((ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3h)\ V1l1))\ V2l2) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V3h)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ A_27a)\ V1l1)\ V2l2))))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (\forall V0P \in (2^{(ty_2Elist_2Elist\ A_27a)}). \\ & (((p\ (ap\ V0P\ (c_2Elist_2ENIL\ A_27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A_27a. (p\ (ap\ V0P\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist\ A_27a). (p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & ((\forall V0x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ (c_2Elist_2ENIL\ A_27a)))) \Leftrightarrow False)) \wedge (\forall V1x \in A_27a. (\forall V2h \in \\ & A_27a. (\forall V3t \in (ty_2Elist_2Elist\ A_27a). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2h)\ V3t)))) \Leftrightarrow ((V1x = V2h) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1x)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ V3t)))))))) \end{aligned} \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption\ A_27a). ((V0opt = (c_2Eoption_2ENONE\ A_27a)) \vee (\exists V1x \in A_27a. (V0opt = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)))))) \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0v \in A_27b. (\forall V1f \in (A_27b^{A_27a}). ((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\ & A_27a. (\forall V3v \in A_27b. (\forall V4f \in (A_27b^{A_27a}). ((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0x \in A_27a. ((p\ (ap\ (c_2Eoption_2EIS_SOME \\ A_27a)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x))) \Leftrightarrow True)) \wedge ((p\ (ap\ (c_2Eoption_2EIS_SOME \\ A_27a)\ (c_2Eoption_2ENONE\ A_27a))) \Leftrightarrow False)) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ (\forall V0v \in A_27b. ((ap\ (ap\ (c_2EpatternMatches_2EPMATCH\ A_27a \\ A_27b)\ V0v)\ (c_2Elist_2ENIL\ ((ty_2Eoption_2Eoption\ A_27a)^{A_27b}))) = \\ (c_2EpatternMatches_2EPMATCH_INCOMPLETE\ A_27a))) \wedge (\forall V1v \in \\ A_27b. (\forall V2r \in ((ty_2Eoption_2Eoption\ A_27a)^{A_27b}). (\forall V3rs \in \\ (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption\ A_27a)^{A_27b})). ((\\ ap\ (ap\ (c_2EpatternMatches_2EPMATCH\ A_27a\ A_27b)\ V1v)\ (ap\ (ap\ (\\ c_2Elist_2ECONS\ ((ty_2Eoption_2Eoption\ A_27a)^{A_27b})\ V2r)\ V3rs)) = \\ (ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27a)\ (ap\ V2r\ V1v)) \\ (ap\ (ap\ (c_2EpatternMatches_2EPMATCH\ A_27a\ A_27b)\ V1v)\ V3rs)) \\ (c_2Ecombin_2EI\ A_27a)))))) \end{aligned} \quad (35)$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0v \in A_27a. (\forall V1rows1 \in (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption \\ A_27b)^{A_27a})). (\forall V2rows2 \in (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption \\ A_27b)^{A_27a})). ((ap\ (ap\ (c_2EpatternMatches_2EPMATCH\ A_27b\ A_27a) \\ V0v)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ ((ty_2Eoption_2Eoption\ A_27b)^{A_27a}) \\ V1rows1)\ V2rows2))) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ (ap\ (c_2Ebool_2E_3F \\ ((ty_2Eoption_2Eoption\ A_27b)^{A_27a}))\ (\lambda V3r \in ((ty_2Eoption_2Eoption \\ A_27b)^{A_27a}). (ap\ (ap\ c_2Ebool_2E_2F_5C\ (ap\ (ap\ (c_2Ebool_2EIN \\ ((ty_2Eoption_2Eoption\ A_27b)^{A_27a})\ V3r)\ (ap\ (c_2Elist_2ELIST_TO_SET \\ ((ty_2Eoption_2Eoption\ A_27b)^{A_27a})\ V1rows1))))\ (ap\ (c_2Eoption_2EIS_SOME \\ A_27b)\ (ap\ V3r\ V0v))))))\ (ap\ (ap\ (c_2EpatternMatches_2EPMATCH \\ A_27b\ A_27a)\ V0v)\ V1rows1))\ (ap\ (ap\ (c_2EpatternMatches_2EPMATCH \\ A_27b\ A_27a)\ V0v)\ V2rows2)))))) \end{aligned}$$