

thm_2EpatternMatches_2EPMATCH__EQUIV__APPEND
(TMST-
gJSeomLB3YVpqHEAktEcFVHsL22YKW9)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 7 We define $c_2Ebool_2E_5C_2E_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (1)$$

Let $c_2Eoption_2EIS_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2EIS_SOME A_27a \in (2^{(ty_2Eoption_2Eoption A_27a)}) \quad (2)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (3)$$

Let $c_2Elist_2EELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EELIST_TO_SET A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist A_27a)}) \quad (4)$$

Definition 8 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A-27a}). (ap\ V1f\ V0x)))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2)))$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \mathbf{if}\ (\exists x \in A. p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x. x \in A) \wedge \iota)$ of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a))))$

Let $c_2EpatternMatches_2EPMATCH : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2EpatternMatches_2EPMATCH \\ A_27a\ A_27b \in ((A_27a^{(ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption\ A_27a)^{A-27b})})^{A_27b}) \end{aligned} \quad (5)$$

Definition 12 We define $c_2EpatternMatches_2EPMATCH_EQUIV_ROWS$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0v \in A_27a. \lambda V1rows1 \in (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption\ A_27b)^{A-27a}))$

Definition 13 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (V0t \Rightarrow V1t1 \wedge V2t2))))$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (6)$$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (9)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A-27a}). (\forall V1Q \in \\ (2^{A-27a}). ((\exists V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ ((\exists V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \vee (\exists V4x \in A_27a. (p\ (ap\ V1Q\ V4x)))))) \end{aligned} \quad (11)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \vee (p V2C)) \wedge (p V0A)) \Leftrightarrow (((p V1B) \wedge (p V0A)) \vee ((p V2C) \wedge (p V0A)))))) \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (13)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow & (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND \ A_27a) \\ & V0P) \ V2x) \ V4y) = (ap (ap (ap (c_2Ebool_2ECOND \ A_27a) \ V1Q) \ V3x_27 \\ & \ V5y_27)))))))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow & (\forall V0e \in A_27a. (\forall V1l1 \in \\ & (ty_2Elist_2Elist \ A_27a). (\forall V2l2 \in (ty_2Elist_2Elist \ A_27a). \\ & ((p (ap (ap (c_2Ebool_2EIN \ A_27a) \ V0e) (ap (c_2Elist_2ELIST_TO_SET \\ & \ A_27a) (ap (ap (c_2Elist_2EAPPEND \ A_27a) \ V1l1) \ V2l2)))) \Leftrightarrow ((p (ap \\ & (ap (c_2Ebool_2EIN \ A_27a) \ V0e) (ap (c_2Elist_2ELIST_TO_SET \\ & \ A_27a) \ V1l1))) \vee (p (ap (ap (c_2Ebool_2EIN \ A_27a) \ V0e) (ap (c_2Elist_2ELIST_TO_SET \\ & \ A_27a) \ V2l2)))))))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow & \forall A_27b.nonempty \ A_27b \Rightarrow (\\ & \forall V0v \in A_27a. (\forall V1rows1 \in (ty_2Elist_2Elist \ ((ty_2Eoption_2Eoption \\ & \ A_27b)^{A_27a})). (\forall V2rows2 \in (ty_2Elist_2Elist \ ((ty_2Eoption_2Eoption \\ & \ A_27b)^{A_27a})). ((ap (ap (c_2EpatternMatches_2EPMATCH \ A_27b \ A_27a) \\ & \ V0v) (ap (ap (c_2Elist_2EAPPEND \ ((ty_2Eoption_2Eoption \ A_27b)^{A_27a}) \\ & \ V1rows1) \ V2rows2)) = (ap (ap (ap (c_2Ebool_2ECOND \ A_27b) (ap (c_2Ebool_2E_3F \\ & \ ((ty_2Eoption_2Eoption \ A_27b)^{A_27a})) (\lambda V3r \in ((ty_2Eoption_2Eoption \\ & \ A_27b)^{A_27a}). (ap (ap \ c_2Ebool_2E_2F_5C (ap (ap (c_2Ebool_2EIN \\ & \ ((ty_2Eoption_2Eoption \ A_27b)^{A_27a})) \ V3r) (ap (c_2Elist_2ELIST_TO_SET \\ & \ ((ty_2Eoption_2Eoption \ A_27b)^{A_27a})) \ V1rows1))) (ap (c_2Eoption_2EIS_SOME \\ & \ A_27b) (ap \ V3r \ V0v)))))) (ap (ap (c_2EpatternMatches_2EPMATCH \\ & \ A_27b \ A_27a) \ V0v) \ V1rows1)) (ap (ap (c_2EpatternMatches_2EPMATCH \\ & \ A_27b \ A_27a) \ V0v) \ V2rows2)))))) \end{aligned} \quad (16)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0v \in A_27a. (\forall V1rows1a \in (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption \\ & A_27b)^{A_27a})). (\forall V2rows1b \in (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption \\ & A_27b)^{A_27a})). (\forall V3rows2a \in (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption \\ & A_27b)^{A_27a})). (\forall V4rows2b \in (ty_2Elist_2Elist\ ((ty_2Eoption_2Eoption \\ & A_27b)^{A_27a})). ((p\ (ap\ (ap\ (ap\ (c_2EpatternMatches_2EPMATCH_EQUIV_ROWS \\ & A_27a\ A_27b)\ V0v)\ V1rows1a)\ V2rows1b)) \Rightarrow ((p\ (ap\ (ap\ (ap\ (c_2EpatternMatches_2EPMATCH_EQUIV_ \\ & A_27a\ A_27b)\ V0v)\ V3rows2a)\ V4rows2b)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2EpatternMatches_2EPMATCH_EQUIV_ \\ & A_27a\ A_27b)\ V0v)\ (ap\ (ap\ (c_2Elist_2EAPPEND\ ((ty_2Eoption_2Eoption \\ & A_27b)^{A_27a}))\ V1rows1a)\ V3rows2a))\ (ap\ (ap\ (c_2Elist_2EAPPEND \\ & ((ty_2Eoption_2Eoption\ A_27b)^{A_27a}))\ V2rows1b)\ V4rows2b)))))))))) \end{aligned}$$