

thm_2EpatternMatches_2EPMATCH__EVAL (TMN4DdWP5dYDSYD69B4PPb3emmJb3pMmKxm)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_7E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 6 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{3}$$

Definition 10 We define c_Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_Esum_2EABS$
Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (5)$$

Definition 11 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a) (c$

Definition 12 We define c_Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_Esum_2EABS$

Definition 13 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_ABS$

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40$

Definition 15 We define $c_2EpatternMatches_2EPMATCH_ROW_COND$ to be
 $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0pat \in (A_27b^{A_27a}). \lambda V1guard \in (2^{A_27a}). \lambda V2inp \in A_27b. \lambda V3v \in A_27a. (ap (ap$

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 17 We define $c_2Eoption_2ESome$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{A_27a}). (ap (ap (ap (c_2Ebool_2ECOND$

Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Eoption_2EOPTION_MAP A_27a A_27b \in (((ty_2Eoption_2Eoption A_27b)^{(ty_2Eoption_2Eoption A_27a)})^{(A_27b^{A_27a})}) \quad (6)$$

Definition 18 We define $c_2EpatternMatches_2EPMATCH_ROW$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V$

Definition 19 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x)$

Definition 20 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 21 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a}) A$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Eoption_2Eoption_CASE A_27a A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption A_27a)}) \quad (7)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (8)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (9)$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ebool_2EARB A_27a \in A_27a \quad (10)$$

Definition 22 We define `c_2EpatternMatches_2EPMATCH_INCOMPLETE` to be $\lambda A_{.27a} : \iota.(c_2Ebool_2EARB A_{.27a})$.

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow c_2Elist_2ENIL A_{.27a} \in (ty_2Elist_2Elist A_{.27a}) \quad (11)$$

Let $c_2EpatternMatches_2EPMATCH : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow \forall A_{.27b}. \text{nonempty } A_{.27b} \Rightarrow c_2EpatternMatches_2EPMATCH A_{.27a} A_{.27b} \in ((A_{.27a}^{(ty_2Elist_2Elist ((ty_2Eoption_2Eoption A_{.27a})^{A_{.27b}}))})^{A_{.27b}}) \quad (12)$$

Definition 23 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. (p V2t))))))$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (15)$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_{.27a}. (p V0t)) \Leftrightarrow (p V0t))) \quad (16)$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_{.27a}. (p V0t)) \Leftrightarrow (p V0t))) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow \\
& True))
\end{aligned} \tag{22}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\
& A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{23}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p V0t))))))
\end{aligned} \tag{24}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0t1 \in A.27a.(\forall V1t2 \in \\
& A.27a.(((ap (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2EF) \\
& V0t1) V1t2) = V1t2))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in \\
& A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\
& 2.(((\forall V2x \in A.27a.(p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in \\
& A.27a.((p (ap V0P V3x)) \wedge (p V1Q))))))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1b \in 2. (\forall V2x \in A_27a. \\
& \quad (\forall V3y \in A_27a. ((ap\ V0f\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\
& \quad V1b)\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ V1b)\ (ap\ V0f \\
& \quad V2x))\ (ap\ V0f\ V3y))))))))) \\
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& \quad (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\
& \quad (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& \quad ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\
& \quad V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\
& \quad V5y_27))))))))) \\
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in \\
& \quad A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (\\
& \quad ap\ V0P\ V1a)))))) \\
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI \\
& \quad A_27a)\ V0x) = V0x)) \\
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\
& \quad A_27a). ((V0opt = (c_2Eoption_2ENONE\ A_27a)) \vee (\exists V1x \in A_27a. \\
& \quad (V0opt = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)))))) \\
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad (\forall V0v \in A_27b. (\forall V1f \in (A_27b^{A_27a}). ((ap\ (ap\ (ap\ (c_2Eoption_2Eoption_CASE \\
& \quad A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in \\
& \quad A_27a. (\forall V3v \in A_27b. (\forall V4f \in (A_27b^{A_27a}). ((ap\ (ap \\
& \quad (ap\ (c_2Eoption_2Eoption_CASE\ A_27a\ A_27b)\ (ap\ (c_2Eoption_2ESOME \\
& \quad A_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \\
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1x \in A.27a. \\
& (\forall V2y \in A.27a. (((ap (ap (ap (c.2Ebool.2ECOND (ty.2Eoption.2Eoption \\
& A.27a)) V0P) (ap (c.2Eoption.2ESOME A.27a) V1x)) (c.2Eoption.2ENONE \\
& A.27a)) = (c.2Eoption.2ENONE A.27a)) \Leftrightarrow (\neg(p\ V0P)))) \wedge (((ap (ap (\\
& ap (c.2Ebool.2ECOND (ty.2Eoption.2Eoption A.27a)) V0P) (c.2Eoption.2ENONE \\
& A.27a)) (ap (c.2Eoption.2ESOME A.27a) V1x)) = (c.2Eoption.2ENONE \\
& A.27a)) \Leftrightarrow (p\ V0P)) \wedge (((ap (ap (ap (c.2Ebool.2ECOND (ty.2Eoption.2Eoption \\
& A.27a)) V0P) (ap (c.2Eoption.2ESOME A.27a) V1x)) (c.2Eoption.2ENONE \\
& A.27a)) = (ap (c.2Eoption.2ESOME A.27a) V2y)) \Leftrightarrow ((p\ V0P) \wedge (V1x = V2y))) \wedge \\
& (((ap (ap (ap (c.2Ebool.2ECOND (ty.2Eoption.2Eoption A.27a)) \\
& V0P) (c.2Eoption.2ENONE A.27a)) (ap (c.2Eoption.2ESOME A.27a) \\
& V1x)) = (ap (c.2Eoption.2ESOME A.27a) V2y)) \Leftrightarrow ((\neg(p\ V0P)) \wedge (V1x = \\
& V2y)))))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in (A.27b^{A.27a}). (\forall V1x \in (ty.2Eoption.2Eoption \\
& A.27a). (\forall V2y \in A.27b. (((ap (ap (c.2Eoption.2EOPTION_MAP \\
& A.27a\ A.27b) V0f) V1x) = (ap (c.2Eoption.2ESOME A.27b) V2y)) \Leftrightarrow (\exists V3z \in \\
& A.27a. ((V1x = (ap (c.2Eoption.2ESOME A.27a) V3z)) \wedge (V2y = (ap\ V0f \\
& V3z)))))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in (A.27a^{A.27b}). (\forall V1x \in (ty.2Eoption.2Eoption \\
& A.27b). (((ap (ap (c.2Eoption.2EOPTION_MAP A.27b\ A.27a) V0f) \\
& V1x) = (c.2Eoption.2ENONE A.27a)) \Leftrightarrow (V1x = (c.2Eoption.2ENONE A.27b))) \wedge \\
& (((c.2Eoption.2ENONE A.27a) = (ap (ap (c.2Eoption.2EOPTION_MAP \\
& A.27b\ A.27a) V0f) V1x)) \Leftrightarrow (V1x = (c.2Eoption.2ENONE A.27b))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& (\forall V0v \in A.27b. ((ap (ap (c.2EpatternMatches.2EPMATCH A.27a \\
& A.27b) V0v) (c.2Elist.2ENIL ((ty.2Eoption.2Eoption A.27a)^{A.27b})) = \\
& (c.2EpatternMatches.2EPMATCH_INCOMPLETE A.27a))) \wedge (\forall V1v \in \\
& A.27b. (\forall V2r \in ((ty.2Eoption.2Eoption A.27a)^{A.27b}). (\forall V3rs \in \\
& (ty.2Elist.2Elist ((ty.2Eoption.2Eoption A.27a)^{A.27b})). ((\\
& ap (ap (c.2EpatternMatches.2EPMATCH A.27a\ A.27b) V1v) (ap (ap (\\
& c.2Elist.2ECONS ((ty.2Eoption.2Eoption A.27a)^{A.27b}) V2r) V3rs)) = \\
& (ap (ap (ap (c.2Eoption.2Eoption_CASE A.27a\ A.27a) (ap V2r V1v)) \\
& (ap (ap (c.2EpatternMatches.2EPMATCH A.27a\ A.27b) V1v) V3rs)) \\
& (c.2Ecombin.2EI A.27a)))))))))
\end{aligned} \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(\forall V3s \in 2.(((p V0p) \Leftrightarrow (ap (ap (ap (c_2Ebool_2ECOND 2) V1q) V2r) V3s)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (\neg(p V3s)))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V3s)))) \wedge (((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))) \wedge ((p V1q) \vee ((p V3s) \vee (\neg(p V0p)))))))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (49)$$

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$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (50)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow \forall A.27c. \\ & nonempty \ A.27c \Rightarrow \forall A.27d.nonempty \ A.27d \Rightarrow (\forall V0v \in A.27b. \\ & (\forall V1p \in (A.27b^{A.27d}). (\forall V2g \in (2^{A.27d}). (\forall V3r \in \\ & (A.27c^{A.27d}). (\forall V4rs \in (ty_2Elist_2Elist \ ((ty_2Eoption_2Eoption \\ & A.27c)^{A.27b})). (((ap \ (ap \ (c_2EpatternMatches_2EPMATCH \ A.27a \\ & A.27b) \ V0v) \ (c_2Elist_2ENIL \ ((ty_2Eoption_2Eoption \ A.27a)^{A.27b}))) = \\ & (c_2EpatternMatches_2EPMATCH_INCOMPLETE \ A.27a)) \wedge ((ap \ (ap \\ & (c_2EpatternMatches_2EPMATCH \ A.27c \ A.27b) \ V0v) \ (ap \ (ap \ (c_2Elist_2ECONS \\ & ((ty_2Eoption_2Eoption \ A.27c)^{A.27b})) \ (ap \ (ap \ (ap \ (c_2EpatternMatches_2EPMATCH_ROW \\ & A.27c \ A.27d \ A.27b) \ V1p) \ V2g) \ V3r)) \ V4rs)) = (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \\ & A.27c) \ (ap \ (c_2Ebool_2E3F \ A.27d) \ (\lambda V5x \in A.27d. (ap \ (ap \ (ap \ (\\ & ap \ (c_2EpatternMatches_2EPMATCH_ROW_COND \ A.27d \ A.27b) \ V1p) \\ & V2g) \ V0v) \ V5x)))) \ (ap \ V3r \ (ap \ (c_2Emin_2E40 \ A.27d) \ (\lambda V6x \in A.27d. \\ & (ap \ (ap \ (ap \ (ap \ (c_2EpatternMatches_2EPMATCH_ROW_COND \ A.27d \\ & A.27b) \ V1p) \ V2g) \ V0v) \ V6x)))))) \ (ap \ (ap \ (c_2EpatternMatches_2EPMATCH \\ & A.27c \ A.27b) \ V0v) \ V4rs))))))))) \end{aligned}$$