thm_2EpatternMatches_2EPMATCH__EVAL__MATCH (TMTz1dUd9qjDx8ULNQjFvTqdN6g4mHDgPD8)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \tag{3}$$

Definition 10 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{4}$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{5}$$

Definition 11 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 12 *We define* c_2EpatternMatches_2EPMATCH__ROW__COND *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0pat \in (A\_27b^{A\_27a}).\lambda V1guard \in (2^{A\_27a}).\lambda V2inp \in A\_27b.\lambda V3v \in A\_27a.(ap\ (ap$

Definition 13 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 14 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 15 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 16 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 17 *We define* c_2Eoption_2Esome *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (ap$ (c_2Ebool_2EC

Let $c\_2Eoption\_2EOPTION\_\_MAP : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{6}$$

Definition 18 *We define* c_2EpatternMatches_2EPMATCH__ROW *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V$

Let $ty\_2Elist\_2Elist : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{7}$$

Let $c\_2Elist\_2ECONS : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{8}$$

Let $c\_2Ebool\_2EARB : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ebool\_2EARB\ A\_27a \in A\_27a \tag{9}$$

Definition 19 *We define* c_2EpatternMatches_2EPMATCH__INCOMPLETE *to be* $\lambda A\_27a : \iota.(c\_2Ebool\_2EARB\ A\_27a)$.

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{10}$$

Let $c\_2EpatternMatches\_2EPMATCH : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))})^{A\_27b}) \tag{11}$$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{13}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg (\forall V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg (p\ (ap\ V0P\ V2x)))))) \tag{19}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{20}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.(\forall V5y\_27 \in A\_27a.((((p\ V0P)\Leftrightarrow(p\ V1Q))\wedge(((p\ V1Q)\Rightarrow(V2x = V3x\_27))\wedge((\neg(p\ V1Q))\Rightarrow(V4y = V5y\_27))))\Rightarrow((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27))))))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c.nonempty\ A\_27c\Rightarrow(\forall V0p \in (A\_27c^{A\_27b}).(\forall V1g \in (2^{A\_27b}).(\forall V2r \in (A\_27a^{A\_27b}).(\forall V3i \in A\_27c.(((ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW}\ A\_27a\ A\_27b\ A\_27c)\ V0p)\ V1g)\ V2r)\ V3i) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\Leftrightarrow(\forall V4x \in A\_27b.(\neg(p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW\_\_COND}\ A\_27b\ A\_27c)\ V0p)\ V1g)\ V3i)\ V4x))))))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c.nonempty\ A\_27c\Rightarrow\forall A\_27d.nonempty\ A\_27d\Rightarrow(\forall V0v \in A\_27b.(\forall V1p \in (A\_27b^{A\_27d}).(\forall V2g \in (2^{A\_27d}).(\forall V3r \in (A\_27c^{A\_27d}).(\forall V4rs \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b})).(((ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27b)\ V0v)\ (c\_2Elist\_2ENIL\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))) = (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_INCOMPLETE}\ A\_27a))\wedge((ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27c\ A\_27b)\ V0v)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption\ A\_27c)^{A\_27b}))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW}\ A\_27c\ A\_27d\ A\_27b)\ V1p)\ V2g)\ V3r))\ V4rs)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27c)\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F}\ A\_27d)\ (\lambda V5x \in A\_27d.(ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW\_\_COND}\ A\_27d\ A\_27b)\ V1p)\ V2g)\ V0v)\ V5x))))\ (ap\ V3r\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A\_27d)\ (\lambda V6x \in A\_27d.(ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW\_\_COND}\ A\_27d\ A\_27b)\ V1p)\ V2g)\ V0v)\ V6x)))))\ (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27c\ A\_27b)\ V0v)\ V4rs))))))))) \tag{24}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0p \in (A\_27c^{A\_27b}).(\forall V1g \in (2^{A\_27b}).(\forall V2r \in (A\_27a^{A\_27b}).(\forall V3v \in A\_27c.(\forall V4rs \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27c})).((\neg((ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW}\ A\_27a\ A\_27b\ A\_27c)\ V0p)\ V1g)\ V2r)\ V3v) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))) \Rightarrow ((ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27c)\ V3v)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27c}))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW}\ A\_27a\ A\_27b\ A\_27c)\ V0p)\ V1g)\ V2r))\ V4rs)) = (ap\ V2r\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A\_27b)\ (\lambda V5x \in A\_27b.(ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW\_\_COND}\ A\_27b\ A\_27c)\ V0p)\ V1g)\ V3v)\ V5x))))))))))$$