thm_2EpatternMatches_2EPMATCH__EXPAND__PRED__THM__G (TMNJgGjGN-QUQoNSz4U9wPaufNnvKsz3coun)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ecombin_2EC *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27c$

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2ELIST\_\_TO\_\_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{2}$$

Definition 4 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27$

Definition 6 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{3}$$

Definition 9 *We define* $\mathsf{c\_2Emin\_2E\_3D\_3D\_3E}$ *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 10 *We define* $\mathsf{c\_2Ebool\_2E\_2F\_5C}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{4}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{5}$$

Definition 11 *We define* $\mathsf{c\_2Esum\_2EINL}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{6}$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{7}$$

Definition 12 *We define* $\mathsf{c\_2Eoption\_2ESOME}$ *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 13 *We define* $\mathsf{c\_2Ecombin\_2EK}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 14 *We define* $\mathsf{c\_2Ecombin\_2ES}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 15 *We define* $\mathsf{c\_2Ecombin\_2EI}$ *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{8}$$

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \tag{9}$$

Definition 16 *We define* $\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_INCOMPLETE}$ *to be* $\lambda A\_27a : \iota.(c\_2Ebool\_2EARB\ A\_27a)$.

Let $c\_2Eoption\_2EIS\_\_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_\_SOME\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{10}$$

Let $c\_2Elist\_2EEXISTS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEXISTS\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \tag{11}$$

Let $c\_2EpatternMatches\_2EPMATCH : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))})^{A\_27b}) \tag{12}$$

Let $c\_2Eoption\_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2ETHE\ A\_27a \in (A\_27a^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{13}$$

Definition 17 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 18 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 19 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 20 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 21 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 22 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 23 *We define* c_2EpatternMatches_2EPMATCH__ROW__COND__NOT__EX__OR__EQ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in A\_27a.\lambda V1r \in ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}).\lambda V2rows \in (ty\_2E$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{14}$$

Definition 24 *We define* c_2EpatternMatches_2EPMATCH__IS__EXHAUSTIVE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0v \in A\_27a.\lambda V1rs \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{15}$$

Let $c\_2EpatternMatches\_2EPMATCH\_\_EXPAND\_\_PRED : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2EpatternMatches\_2EPMATCH\_\_EX\ldots A\_27a\ A\_27b \in ((((2^{(ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))})^{(ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ \ldots} \tag{16}$$

Let $c\_2Elist\_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EREVERSE\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{17}$$

Let $c\_2Elist\_2EEVERY : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEVERY\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \tag{18}$$

Assume the following.

$$True \tag{19}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \lor (\neg (p\ V0t)))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True \land (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \land True) \Leftrightarrow (p\ V0t)) \land (((False \land (p\ V0t)) \Leftrightarrow False) \land ((((p\ V0t) \land False) \Leftrightarrow False) \land (((p\ V0t) \land (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(((True \lor (p\ V0t)) \Leftrightarrow True) \land ((((p\ V0t) \lor True) \Leftrightarrow True) \land (((False \lor (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \lor False) \Leftrightarrow (p\ V0t)) \land (((p\ V0t) \lor (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{26}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg (\forall V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg (p\ (ap\ V0P\ V2x)))))) \tag{32}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.((\neg ((p\ V0A) \Rightarrow (p\ V1B))) \Leftrightarrow ((p\ V0A) \wedge (\neg (p\ V1B)))))) \tag{33}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p\ V1B) \wedge (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A))))))) \tag{34}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{35}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow(((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \quad (36)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \quad (37)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow((\forall V0P \in (2^{A\_27a}).((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V0P)\ (c\_2Elist\_2ENIL\ A\_27a)))\Leftrightarrow True))\wedge(\forall V1P \in (2^{A\_27a}).(\forall V2h \in A\_27a.(\forall V3t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V1P)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V3t)))\Leftrightarrow((p\ (ap\ V1P\ V2h))\wedge(p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V1P)\ V3t)))))))) \quad (38)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}).(((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a)))\wedge(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ V0P\ V1t))\Rightarrow(\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t)))))))\Rightarrow(\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ V0P\ V3l))))) \quad (39)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V0P)\ V1l))\Leftrightarrow(\forall V2e \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2e)\ (ap\ (c\_2Elist\_2ELIST\_\_TO\_\_SET\ A\_27a)\ V1l)))\Rightarrow(p\ (ap\ V0P\ V2e))))))) \quad (40)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((\neg(p\ (ap\ (ap\ (c\_2Elist\_2EEXISTS\ A\_27a)\ V0P)\ V1l)))\Leftrightarrow(p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ 2\ 2)\ \mathsf{c\_2Ebool\_2E\_7E})\ V0P))\ V1l))))) \quad (41)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0opt \in (ty\_2Eoption\_2Eoption\ A\_27a).((V0opt = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\vee(\exists V1x \in A\_27a.(V0opt = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))))) \quad (42)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg((\mathsf{c\_2Eoption\_2ENONE}\ A\_27a) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x)))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (c\_2Eoption\_2ETHE\ A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x)) = V0x)) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Eoption\_2Eoption\ A\_27a).((\neg(p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ V0x))) \Leftrightarrow (V0x = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0v \in A\_27b.((ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27b)\ V0v)\ (c\_2Elist\_2ENIL\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))) = (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_INCOMPLETE}\ A\_27a))) \land (\forall V1v \in A\_27b.(\forall V2r \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}).(\forall V3rs \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})).((ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27b)\ V1v)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))\ V2r)\ V3rs)) = (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27a)\ (ap\ V2r\ V1v))\ (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27b)\ V1v)\ V3rs))\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a))))))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0i \in A\_27a.(\forall V1r \in ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}).(\forall V2r\_27 \in ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}).(\forall V3rows \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a})).((\neg((ap\ V2r\_27\ V0i) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b))) \Rightarrow ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW\_\_COND\_\_NOT\_\_EX\_\_OR\_\_EQ}\ A\_27a\ A\_27b)\ V0i)\ V1r)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}))\ V2r\_27)\ V3rows))) \Leftrightarrow ((\neg((ap\ V1r\ V0i) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b))) \Rightarrow ((ap\ V1r\ V0i) = (ap\ V2r\_27\ V0i))))))))) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0i \in A\_27a.(\forall V1r \in ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}). ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW\_\_COND\_\_NOT\_\_EX\_\_OR\_\_EQ}\ A\_27a\ A\_27b)\ V0i)\ V1r)\ (c\_2Elist\_2ENIL\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}))))\Leftrightarrow((\neg((ap\ V1r\ V0i) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b)))\Rightarrow False)))) \tag{49}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0i \in A\_27a.(\forall V1r\_27 \in ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}). (\forall V2rows \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a})). (\forall V3r \in ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW\_\_COND\_\_NOT\_\_EX\_\_OR\_\_EQ}\ A\_27a\ A\_27b)\ V0i)\ V1r\_27)\ V2rows))\Rightarrow((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW\_\_COND\_\_}\ A\_27a\ A\_27b)\ V0i)\ V3r)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption\ A\_27b)^{A\_27a}))\ V1r\_27)\ V2rows)))\Leftrightarrow(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW\_\_COND\_\_N}\ A\_27a\ A\_27b)\ V0i)\ V3r)\ V2rows)))))))) \tag{50}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1v \in A\_27b.(\forall V2r \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}).(\forall V3rows \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})).((p\ (ap\ V0P\ (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27b)\ V1v)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))\ V2r)\ V3rows))))\Leftrightarrow(((\neg((ap\ V2r\ V1v) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)))\Rightarrow(p\ (ap\ V0P\ (ap\ (c\_2Eoption\_2ETHE\ A\_27a)\ (ap\ V2r\ V1v)))))\land ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW\_\_COND\_\_NOT\_\_EX\_\_OR\_\_EQ}\ A\_27b\ A\_27a)\ V1v)\ V2r)\ V3rows))\Rightarrow(p\ (ap\ V0P\ (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27b)\ V1v)\ V3rows)))))))))) \tag{51}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0P \in (2^{A\_27a}).(\forall V1v \in A\_27b.(\forall V2rows\_\_before \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})).((p\ (ap\ (ap\ (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\_\_EXPAND\_\_PRED\ A\_27a\ A\_27b)\ V0P)\ V1v)\ V2rows\_\_before)\ (c\_2Elist\_2ENIL\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))))\Leftrightarrow((\neg(p\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_IS\_\_EXHAUSTIVE}\ A\_27b\ A\_27a)\ V1v)\ (ap\ (c\_2Elist\_2EREVERSE\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))\ V2rows\_\_before))))\Rightarrow(p\ (ap\ V0P\ (c\_2Ebool\_2EARB\ A\_27a))))))))\wedge(\forall V3P \in (2^{A\_27a}).(\forall V4v \in A\_27b.(\forall V5rows\_\_before \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})).(\forall V6r \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}). (\forall V7rows\_\_after \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})).((p\ (ap\ (ap\ (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\_\_EXPAND\_\_PRED\ A\_27a\ A\_27b)\ V3P)\ V4v)\ V5rows\_\_before)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))\ V6r)\ V7rows\_\_after)))\Leftrightarrow (((\neg((ap\ V6r\ V4v) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)))\Rightarrow((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))\ (\lambda V8r\_27 \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}).(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ (ap\ \mathsf{c\_2Ebool\_2E\_7E}\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ V8r\_27\ V4v))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ (ap\ V8r\_27\ V4v))\ (ap\ V6r\ V4v)))))\ V5rows\_\_before))\Rightarrow(p\ (ap\ V3P\ (ap\ (c\_2Eoption\_2ETHE\ A\_27a)\ (ap\ V6r\ V4v))))))\wedge(p\ (ap\ (ap\ (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\_\_EXPAND\_\_PRED\ A\_27a\ A\_27b)\ V3P)\ V4v)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))\ V6r)\ V5rows\_\_before))\ V7rows\_\_after)))))))))) \tag{52}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1l \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V0P)\ (ap\ (c\_2Elist\_2EREVERSE\ A\_27a)\ V1l)))\Leftrightarrow(p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ A\_27a)\ V0P)\ V1l))))) \tag{53}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{54}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{55}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow (((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{56}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{57}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{58}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee ((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{59}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee (\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge ((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{62}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{63}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{64}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{65}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{66}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{67}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{68}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1v \in A\_27b.(\forall V2rows\_\_before \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})).(\forall V3rows\_\_after \in (ty\_2Elist\_2Elist\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})).((p\ (ap\ (ap\ (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\_\_EXPAND\_\_PRED\ A\_27a\ A\_27b)\ V0P)\ V1v)\ V2rows\_\_before)\ V3rows\_\_after))\Leftrightarrow((p\ (ap\ (ap\ (c\_2Elist\_2EEVERY\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}))\ (\lambda V4r \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b}).(ap\ (ap\ (ap\ (\mathsf{c\_2EpatternMatches\_2EPMATCH\_\_ROW\_\_COI}\ A\_27b\ A\_27a)\ V1v)\ V4r)\ V3rows\_\_after)))\ V2rows\_\_before))\Rightarrow(p\ (ap\ V0P\ (ap\ (ap\ (c\_2EpatternMatches\_2EPMATCH\ A\_27a\ A\_27b)\ V1v)\ V3rows\_\_afte$$