

thm_2EpatternMatches_2EPMATCH__EXTEND__BASE (TMUzKxqhfLyjis7EJeueTBZyVvb9GY9eKx)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 4 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 5 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (1)$$

Let $c_2Eoption_2Eoption_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Eoption_2Eoption_CASE A_27a A_27b \in (((A_27b^{(A_27b^{A_27a})})^{A_27b})^{(ty_2Eoption_2Eoption A_27a)}) \quad (2)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (3)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (4)$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2EARB A_27a \in A_27a \quad (5)$$

Definition 6 We define $c_EpatternMatches_2EPMATCH_INCOMPLETE$ to be $\lambda A_27a : \iota.(c_2Ebool_2EARB A_27a)$.

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (6)$$

Let $c_2EpatternMatches_2EPMATCH : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2EpatternMatches_2EPMATCH A_27a A_27b \in ((A_27a^{(ty_2Elist_2Elist ((ty_2Eoption_2Eoption A_27a)^{A_27b}))})^{A_27b}) \quad (7)$$

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & (\forall V0v \in A_27b.((ap (ap (c_2EpatternMatches_2EPMATCH A_27a A_27b) V0v) (c_2Elist_2ENIL ((ty_2Eoption_2Eoption A_27a)^{A_27b}))) = \\ & (c_2EpatternMatches_2EPMATCH_INCOMPLETE A_27a))) \wedge (\forall V1v \in \\ & A_27b.(\forall V2r \in ((ty_2Eoption_2Eoption A_27a)^{A_27b}).(\forall V3rs \in \\ & (ty_2Elist_2Elist ((ty_2Eoption_2Eoption A_27a)^{A_27b})).((\\ & ap (ap (c_2EpatternMatches_2EPMATCH A_27a A_27b) V1v) (ap (ap (\\ & c_2Elist_2ECONS ((ty_2Eoption_2Eoption A_27a)^{A_27b}) V2r) V3rs)) = \\ & (ap (ap (ap (c_2Eoption_2Eoption_CASE A_27a A_27a) (ap V2r V1v)) \\ & (ap (ap (c_2EpatternMatches_2EPMATCH A_27a A_27b) V1v) V3rs)) \\ & (c_2Ecombin_2EI A_27a)))))) \end{aligned} \quad (11)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\ & \text{nonempty } A_27c \Rightarrow (\forall V0v_old \in A_27a. (\forall V1v_new \in A_27b. \\ & ((\text{ap } (\text{ap } (\text{c_2EpatternMatches_2EPMATCH } A_27c } A_27a) V0v_old) \\ & (\text{c_2Elist_2ENIL } ((\text{ty_2Eoption_2Eoption } A_27c)^{A_27a}))) = (\text{ap} \\ & (\text{ap } (\text{c_2EpatternMatches_2EPMATCH } A_27c } A_27b) V1v_new) (\text{c_2Elist_2ENIL} \\ & ((\text{ty_2Eoption_2Eoption } A_27c)^{A_27b})))))) \end{aligned}$$